

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN
MATHEMATICS

MATH 326: METHODS OF APPLIED MATHS 1

STREAMS:

TIME: 2 HOURS

DAY/DATE: THURSDAY 13/04/2023

11.30 A.M. – 2.30 P.M.

INSTRUCTIONS

Answer question one and any other two questions
Adhere to the instructions on the answer booklet.

QUESTION ONE Compulsory.

- a. Identify the nature of the singular points of the equation $4x^2y'' + 5xy' + \frac{1}{2}(x-1)y = 0$ (5 marks)
- b. Find the series solutions about $x=0$ of $y'' + y = 0$ (6 marks)
- c. Decompose $\frac{s+3}{(s-2)(s-3)}$ into partial fractions hence evaluate $L^{-1}\left(\frac{s+3}{(s-2)(s-3)}\right)$ (5 marks)
- d. Find the sine Fourier series for the function $f(x)=1$, in $0 < x < \pi$ (5 marks)
- e. Determine the nature of the singular points of the differential equation $\frac{3}{2}xy'' + y' + \frac{1}{2}y = 0$, Hence
Find the roots of its indicial equation (6 marks)
- f. Evaluate the $L^{-1}\left[\frac{1}{s(s^2+4)}\right]$ (5 marks)

QUESTION TWO

- a. The Legendre's equation has the form $(1-z^2)y'' - 2zy' + l(l+1)y = 0$, where l is a constant and z is the dependent variable,

- i. Show that , $z=0$ is a an ordinary point and $z=\pm 1$ is a regular singular point of the equation (5 marks)
- ii. Show that the Legendre`s equation has a regular singularity as $|z|=\infty$ (7 marks)
- b. A periodic function $f(t)$ of period 2π is defined by $f(t)=t^2+t, -\pi < t < \pi$. Evaluate b_n, a_0 and a_n and obtain the Fourier series expansion of the function (8 marks)

QUESTION THREE

- a. Using the Laplace transforms, to evaluate $\int_0^{\infty} t e^{-3t} \sin t dt$ (5 marks)
- b. Given the Bessel`s differential equation $x^2 y'' + xy' + (x^2 - n^2)y = 0$, about the point $x = 0$.
- i. Obtain the roots of the indicial equation of the differential equation (7 marks)
- ii. Find the recurrence relation satisfied by coefficients in the series solution of the differential equation and obtain a_2 (5 marks)
- c. Solve the initial value problem $y' + y = 1, y(0) = 1$ by La[lace transforms (3 marks)

QUESTION FOUR

- a. Solve the system below by Laplace transforms (5 marks)
- $$\begin{aligned} y'' + z + y &= 0 \\ z' + y' &= 0 \end{aligned}$$
- Given*
 $y(0) = 0, y'(0) = 0, z(0) = 1$
- b. Obtain a_0 and a_n and b_n for the Fourier series represented by $f(x) = \begin{cases} 2, & -2 < x < 0 \\ x, & 0 < x < 2 \end{cases}$ (8 marks)
- c. Applying Laplace transform, find the solution of the differential equation $y'' + y = \sin t$, satisfying the initial condition $y(0) = 1, y'(0) = 0$ (7 marks)

QUESTION FIVE

- a. Obtain the Fourier series expansion of the rectified sine wave $f(t) = |\sin t|$ (5 marks)
- b. Evaluate the laplace transform of $t e^{-t} \sin 2t$ (5 marks)
- c. Identify the nature of the singular points of the equation
- $$3x(x-2)^2 y'' + 6(x-2)y' + 3(x+3)y = 0$$
- (6 marks)
- d. Find the Laplace transform of $\frac{\sin 2t}{t}$ (4 marks)