CHUKA  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF DOCTORATE IN PHYSICS

PHYS 941: CLASSICAL ELECTRODYNAMICS AND ELECTROMAGNETIC THEORY

STREAMS: **TIME: 3 HOURS**

DAY/DATE: WEDNESDAY 18/12/2024 **2.30 P.M – 5.30 P.M**

INSTRUCTIONS:

i. **Answer four Questions**

ii. **Do not write on the Question paper**

QUESTION ONE (15 MARKS)

A point charge q is placed at a distance a from the center of a grounded conducting sphere of radius R. The electric potential outside the sphere can be calculated using the method of images.

a) List the boundary conditions that the electric potential V must satisfy on the surface of the sphere and at infinity. **(3 marks)**

b) By calculating the electric potential on the boundaries, show that an image charge located at a distance $b = \frac{R^2}{a}$ from the center of the sphere with charge $q^{'} = -\, q\frac{R}{a}$ creates an electric potential that satisfies the boundary conditions. **(12 marks)**

QUESTION TWO (15 MARKS)

The potential $\phi(x)$ due to a charge distribution $\rho(x')$ in free space can be derived using Green's function. The Green function is defined as the solution to the Poisson equation for a point charge source.

a. Show that the Green function $G\left(x, x^{'}\right) = -\ \frac{1}{\left|x^{'} - x\right|}$ satisfies the following differential equation: $\nabla^2 G\left(x, x^{'}\right) = -\ 4\pi\, \delta\,(x^{'} - \ x)$ **(4 marks)**

b. Explain the significance of the Dirac delta function $\delta(x - x')$ in the context of this equation. **(2 marks)**

c. Using the Green function, express the potential $\phi(x)$ in terms of the charge distribution $\rho(x')$ by solving the Poisson equation: $\nabla^2\phi(x) = -\ 4\pi\rho\left(x^{'}\right)$.Derive the integral form of

the potential $\phi(x)$. **(2 marks)**

d. Discuss the physical interpretation of the Green function $G\left(x, x^{'}\right)$ in electrostatics, and how a point charge located at $x^{'}$ affects the potential at a point x. **(2 marks)**

e. Consider a continuous charge distribution $\rho(x')$. Explain how the total potential $\phi(x)$ point x is obtained by summing the contributions from all charge elements. **(2 marks)**

QUESTION THREE (15 MARKS)

Using Maxwell's equations, analyze the propagation of a plane electromagnetic wave in a source-free conductive medium.

a) Derive the wave equation for the electric field $\nabla^2\vec{E} - \mu\sigma\dot{\vec{E}} - \mu\varepsilon\ddot{\vec{E}} = 0$. Clearly define each term in your derivation. **(3 marks)**

b) What physical insights can be drawn from this wave equation regarding how electromagnetic waves behave in a conductive medium? Discuss any implications of the conductivity σ.

(1 mark)

c) Assume a solution of the form $\vec{E}\left(\vec{r}, t\right) = \vec{E}e^{i(\omega t - \vec{k}.\vec{r})}$. Show that, when considering a complex dispersion relation, the general solution can be expressed as:

$$\vec{E}e^{i\left(\omega t - \vec{\alpha}.\vec{r}\right)}e^{-\gamma.\vec{r}}$$

where α and γ are related to the wave characteristics in the conducting medium. ***(10 marks)***

d) Describe the significance of the two exponential terms in your solution from part (c) and explain how they relate to the attenuation and phase of the electromagnetic wave. ***(1 mark)***

QUESTION FOUR (15 MARKS)

A conducting sphere of radius a has a total charge Q uniformly distributed on its surface. A point charge qqq is placed at a distance y from the center of the sphere, where $y > a$.

i. Calculate the electric potential $\Phi(x)$ at a point x outside the sphere due to both the point charge q and the charge Q on the sphere. **(5 marks)**

ii. Determine the surface charge density σ on the conducting sphere due to the presence of the point charge q. **(3 marks)**

iii. Calculate the net force F_q acting on the charge q due to the conducting sphere**. (7 marks)**

QUESTION FIVE (15 MARKS)

a. A fixed electric charge density $\rho(x)$ exists in space with no boundaries. Write an expression for the electrostatic potential that it produces at a field point x. (**5 marks**)

b. Write an expression that gives the electrostatic field energy density in vacuum. (**5 marks**)

c. From a solution for electric potential $\Phi(x)$, how can you obtain the surface charge density σ on a conductor boundary? Let $\hat{n}$ be the unit vector pointing perpendicularly outward from the conductor. (**5 marks**)
