CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION SCIENCE/ARTS, BACHELOR OF SCIENCE AND BACHELOR OF ARTS (MATH-ECONS)

MATH 302/312: REAL ANALYSIS I

STREAMS: BSC **TIME: 2 HOURS**

DAY/DATE: MONDAY 28/08/2023 **2.30 P.M. – 4.30 P.M**

INSTRUCTIONS:

- **Answer ALL questions.**
- **Do not write on the question paper.**

QUESTION ONE: (30 MARKS)

a) If x_0 is a real number in a set N and $\alpha > 0$ then define the following

i. α- neighbourhood of x_0. (3 marks)

ii. An interior of point of set N. (3 marks)

b) Explain what is meant by saying that a number r is rational. Show that if $p = \sqrt{l+1} - \sqrt{l-1}$ for any integer $l \geq 1$ then p is irrational. (6 marks)

c) Prove that the union of open set is open. (4 marks)

d) Determine whether the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ convergence or diverges. (4 marks)

e) Consider three sets S, T, V show that $U/(SUT) = (V/S) \cap (V/T)$. (5 marks)

f) Show that the sequence $\left\{\frac{1}{2^n}\right\}$ is a Cauchy sequence. (5 marks)

QUESTION TWO: (20 MARKS)

a) (aConsider the set $\mathbb{R}^n$ for all $x \in \mathbb{R}^n, x = x_i$. Define$\mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ by $d(x, y) = Max\{|x_i - y_i|: (i = 1,2, \ldots n)\}$. Prove that $(\mathbb{R}^n, d)$ is a metric space. (6 marks)

.

b) Given the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$, find the partial sum S_n and show that the series converges.

(5 marks)

c) Prove that a function which is uniformly continuous on an interval is continuous on that interval. (4 marks)

d) Prove that every convergent sequence is a Cauchy sequence but the converse need not to be true. (5 marks)

QUESTION THREE: (20 MARKS)

a) Given $f(x) = x^2 - 5x$ show that the $\lim_{x \to 2} f(x) = -6$ and hence determine a value for $\delta > 0$ associated with $\varepsilon > 0$ in accordance with the definition of a limit of a function.

(6 marks)

b) Prove that every convergent sequence is bounded and with the help of an example show that the converse is not necessarily true. (9 marks)

Let $\{y_n\}$ be a sequence of real numbers. Prove that if $\{y_n\}$ converges then its limit is unique. (5 marks)
