CHUKA **UNIVERSITY**

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF ART

MATH 452: TESTING HYPOTHESIS

STREAMS: BSC, BA **TIME: 2 HOURS**

DAY/DATE: TUESDAY 19/12/2023 **8.30 A.M. – 10.30 A.M.**

INSTRUCTIONS:

- Answer question **ONE** and any other **TWO** questions

QUESTION ONE (30 MARKS)

(a). Distinguish between the following terms as used in hypothesis testing

(i). A statistical hypothesis and a test of a statistical hypothesis $(2Marks)$

(ii). Type 1 and type 11 errors $(2Marks)$

(b). After testing a statistical hypothesis, one is bound to arrive at only one of four possible decisions.

(i). State the four decisions $(4Marks)$

(ii). Which of the decisions are correct decisions and which are erroneous decisions? $(2Marks)$

(c). A coin is tossed four times. Suppose the hypothesis $H_0: \theta = \frac{1}{4}$ against $H_1: \theta = \frac{5}{6}$ is to be tested, where θ is the probability that head turns up in a single toss of the coin. Obtain the size and the power of the test if the null hypothesis is to be rejected if 3 or more heads are obtained. $(7Marks)$

(d). The weight of a new package of salt is known to have a normal distribution with a variance of 25. A sample of 8 packages yielded the following weights

43,39,40,42,38,39,36 and 38. Test he hypothesis that the population mean is greater than 40g. Take α to be 5% $(4Marks)$

(e). Suppose two samples of size 8 and 9 respectively are selected from two normally distributed populations with variances σ_1^2 and σ_2^2. Suppose the respective sample standard deviations are found to be $S_1 = 5.2$ and $S_2 = 3.9$. Test the hypothesis that $\sigma_1^2 = \sigma_2^2$ against the two-sided alternative at 5% level of significance. $(5\ Marks)$

(f). The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} \frac{1}{\theta}, 0 \leq x \leq \theta \\ 0\,, elsewhere \end{cases}$$

The hypothesis $H_0: \theta = 1$ is to be tested against the alternative $H_1: \theta = 2$ by means of a single observed value of X. Determine the size of type 11 error using $0.5 \leq x$ as the critical region $(4Marks)$

QUESTION TWO (20 MARKS)

(a). (i). State the Neyman-Pearson lemma $(3\ Marks)$

(ii). Prove the Neyman-Pearson lemma $(7Marks)$

(b). Suppose X has a normal distribution with known mean μ_0 and unknown variance σ^2. Let $x_1,\ x_2, \ldots, x_n$ be a random sample of size n taken from X for testing $H_0: \sigma = \sigma_0$ against $H_1: \sigma = \sigma_1$. Show that the most powerful size α test for testing H_0 against H_1 has the best size α critical region C_α given by;

$$C_\alpha = \left\{x: \sum_{i=1}^{n} (x_i - \mu_0\ )^2 > \sigma_0^2 \chi_{1-\alpha}^2\right\}$$

Where $\sigma_1 > \sigma_0$ $(10\ Marks)$

QUESTION THREE (20 MARKS)

$(a).(i)$. Consider a random sample of size n from a population which is normally distributed with mean μ and variance σ^2 where σ^2 is known. Derive the size α best critical region for testing $H_0: \mu = \mu_0$ against $H_1: \mu = \mu_1$,where μ_0 and μ_1 have specified values and $\mu_1 > \mu_0$ $(11\ Marks)$

(ii). Determine the power of the test corresponding to this critical region $(4\ Marks)$

(i). Test the hypothesis $H_0: \mu = 3$ against $H_1: \mu = 4$ if a random sample from $N(\mu, 25)$ has the values 3,2,9,10,5,6,12,11,2. Take $\alpha = 5\%$ $(5\ Marks)$

QUESTION FOUR (20 MARKS)

(a). Let $x_1,\ x_2, . . . ,x_m$ be a random sample from a normal population with mean μ_1, and variance σ^2 and $y_1,\ y_2, . . . ,y_n$ be another independent random sample from a normal population with mean μ_2, and variance σ^2**.** Derive the test statistic for testing the hypothesis

$H_0: \mu_1 = \mu_2$ against $H_1: \mu_1 \neq \mu_2$ $(12\ Marks)$

(b). The sample 10,30,20,50,20,60,40,50 is from a normal population. Also the sample 20,70,60,80,20,40,20,10,20,30 is from a normal population. Assuming the two samples are independent and the two populations have the same variance, test the hypothesis

$H_0: \mu_1 = \mu_2$ against $H_1: \mu_1 \neq \mu_2$ at 5% level of significance $(8\ Marks)$

QUESTION FIVE (20 MARKS)

$(a).(i)$. Let $(x_1, y_1), (x_2, y_2), ... , (x_n, y_n)$ be a random sample of size n from the bivariate normal distribution. State a suitable test for testing $H_0: \rho = 0$ against $H_1: \rho \neq 0$, where ρ is the correlation coefficient between X and Y. $(1\ Mark)$

(iI). Use the test statistic stated in $(a)(i)$ above to test the hypothesis $H_0: \rho = 0$ against H_1: $\rho \neq 0$,at 5% level of significance if the observations are

(36,25),(60,37),(22,66),(19,26),(43,36) $(12\ Marks)$

(b). Independent random samples were selected from each of two normally distributed populations with $n_1 = 16$ from population 1 and $n_2 = 25$ from population 2. The means and variances of the two samples are as shown below

Sample 1	Sample 2
$n_1 = 16$	$n_2 = 25$
$\bar{x}_1 = 2.25$	$\bar{x}_2 = 28.2$
$s_1^2 = 2.87$	$s_2^2 = 9.85$

Test the hypothesis $H_0: \sigma_1^2 = \sigma_2^2$ against $H_1: \sigma_1^2 \neq \sigma_2^2$ at $\alpha = 5\%$ $(7\ Marks)$

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