CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS STATISTICS, ECONOMICS MATHEMATICS, BACHELOR OF IN MATHEMATICS, BACHELOR OF SCIENCE, BACHELOR OF EDUCATION, BACHELOR OF SCIENCE GENERAL AND BACHELOR OF ARTS (GENERAL)

MATH 344: THEORY OF ESTIMATION

STREAMS: As above **TIME: 2 HOURS**

DAY/DATE: WEDNESDAY 20/12/2023 **2.30 P.M. – 4.30 P.M.**

INSTRUCTIONS:

- Answer question **ONE** and **TWO** other questions

Question One [30 marks]

a) Define the following terms as used in theory of Estimation

(i) Mean squared error consistency

(ii) Consistent estimator

(iii) Unbiased estimator

(iv) Efficient statistic [8 marks]

b) Let $x_i, i = 1,2,3,4$, be four independent sample observations of Poisson distribution with parameter θ. Show that $T = \frac{1}{15}(2x_1 + 4x_2 + 5x_3 + 3x_4)$ is a biased estimator of θ. Calculate the amount of bias. [3 marks]

c) Let T_1 be the most efficient estimator and T_2 be the unbiased estimator for unknown parameter θ. If e is the efficiency with respect to T_1, show that

$Var(T_1 - T_2) = \frac{1-e}{e}\, var(T_1)$ [6 marks]

d) Find sufficient statistic for δ^2 where $x \sim N(\mu, \delta^2)$ [5 marks]

e) The Poisson distribution has two unbiased estimators of the same parameter θ. Show that there exists an infinite number of unbiased estimators of the same parameter θ.

[6 marks]

QUESTION 2 [20 MARKS]

a) Consider two random samples $x_1, x_2, \dots, x_{n1}$ of size n1 and $y_1, y_2, \dots, y_{n2}$ of size n2 both from normal populations such that $x \sim N(\mu_1, \sigma_1^2)$ and $y \sim N(\mu_2, \sigma_2^2)$ respectively. Obtain the $(1-\alpha)100\%$ confidence interval for $(\mu_1 - \mu_2)$. [9 marks]

b) The distribution of $x$is given by $f(x) = \begin{cases} \theta^x (1-\theta)^{1-x} & ,x = 0,1 \\ 0 & elsewhere \end{cases}$

Show that $T = \sum x_i$ is a sufficient statistic for θ. [11 marks]

QUESTION 3 [20 MARKS]

a) Define a uniformly minimum variance unbiased estimator (UMVUE) T of $\tau(\theta)$.

[4 marks]

b) If T is a consistent estimator of θ, $\emptyset(T)$ is also a consistent estimator of $\emptyset(\theta)$ where $\emptyset$ is a continuous function, Proof. [10 marks]

c) Find the lower of variance of T which is an unbiased estimator of θ. [6 marks]

QUESTION 4 [20 MARKS]

a) Let $x_1, x_2, \dots, x_n$ be a random sample of size n taken from the distribution given by

$f(x, \theta) = \theta x^{\theta-1}, \quad 0 \leq x \leq 1, \theta > 0$

If Z=-$\sum_{i=1}^{n} log x_i$, show that $T = \frac{n-1}{Z}$ is an unbiased estimator of θ. [10 Marks]

b) Let T_1 be MVUE of θ, while T_2 is unbiased estimator of θ. If e is the efficiency of T_2 with respect to T_1, show that the $corr(T_1, T_2) = \sqrt{e}$. [10 Marks]

QUESTION 5 [20 MARKS]

a) Let $x_1, x_2, \dots, x_n$ be a random sample of size n taken from a Normal distribution with mean zero and variance θ. Show that the estimator T is an unbiased estimator of θ.

$T = \frac{1}{2(n-1)} \sum_{i=1}^{n-1} (x_{i+1} - x_i)^2$ [8 marks]

b) Let $\bar{x}$ denote the mean of a random sample of size n taken fro a normal population with mean μ and variance 16. Find the sample size such that

$P[\bar{x} - 1 < \mu < \bar{x} + 1] = 0.8664$ [7 marks]

c) Let X be a binomial variate with parameters n and p. Find the estimator of p by methods of moments. [5 marks]

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