

CHUKA



UNIVERSITY

Knowledge is wealth (*Sapientia divitia est*)

**UNIVERSITY EXAMINATIONS
FIRST YEAR EXAMINATION FOR THE AWARD OF MASTERS OF SCIENCE IN
APPLIED MATHEMATICS**

MATH 827: METHODS OF FLUID MECHANICS

TIME: 3 HOURS

DAY/DATE:

INSTRUCTIONS:

Answer Any Three Questions

QUESTION ONE (20 MARKS)

a) Suppose $f(z)$ and $h(z)$ are functions of the continuous complex variable z defined on some domain $D \subset \mathbb{C}$ and possess limits as $z \rightarrow z_0$ in D , define the following terms

- i) Asymptotically bound (2 marks)
- ii) Asymptotically smaller (2 marks)
- iii) Asymptotically equal (2 marks)

b) Given that $f(n) = 3n^2 - 7n + 8$, show that $f(n) = O(n^3)$ as $n \rightarrow \infty$ (4 marks)

c) Given that $f(t) = 5t^2 + t + 3$, show that $f(t) = o(t^3)$ as $t \rightarrow \infty$ (4 marks)

d) Use binomial expansion, obtain the exact solution of the regular perturbation problem $P(\varepsilon)$.

$$f(x, \varepsilon) = x^2 + \varepsilon x - 1 = 0, \text{ where } 0 < \varepsilon \ll 1 \quad (6 \text{ marks})$$

QUESTION TWO (20 MARKS)

a) Distinguish between a regular perturbation and an irregular perturbation problem (4 marks)

b) Obtain the first four terms in power series approximations for the roots of $x^3 - x^2 - (1 + \varepsilon)x + 1 = 0$, near $x=1$, $\varepsilon \ll 1$ (10 marks)

Where ε is a small parameter.

c) Evaluate the exact solution of the singular perturbation problem $P(\varepsilon) = \varepsilon x^2 + x - 1 = 0$, by binomial expansion.

QUESTION THREE (20 MARKS)

- a) Given the function $f(x, y) = x^3 + 3y - y^3 - 3x$, find the Taylor series expansion at point (2,1) (5marks)
- b) Distinguish between an asymptotic sequence and an asymptotic expansion (4marks)
- c) Outline the properties of asymptotic expansion (6marks)
- d) Solve the initial value problem

$$\varepsilon \frac{dx}{dt} + x = 0 \quad \text{with } x(0)=1 \quad (5\text{marks})$$

QUESTION FOUR (20 MARKS)

- a) Solve the initial value problem (5 marks)

$$\varepsilon(x) = Ax(t) + b(t)$$

- b) Find the singular points of differential equation (6 marks)

$$2x^2 y'' + 3xy' + (x^2 - 4)y = 0$$

- c) Solve the series of equation (9 marks)

$$xy'' + y' - y = 0$$

QUESTION FIVE (20 MARKS)

- a) Write down the main principle behind LU decomposition methods (6marks)
- i) Doolittle decomposition
 - ii) Crout decomposition
 - iii) Chelasky factorisation

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 9 & 6 \\ 3 & 6 & 27 \end{bmatrix} \quad \text{and} \quad \tilde{b} = \begin{bmatrix} 2 \\ -4 \\ 24 \end{bmatrix}; \quad \tilde{A}\tilde{X} = \tilde{b} \quad (10 \text{ marks})$$

- b) Consider the matrix A given by $\tilde{A}\tilde{X} = \tilde{b}$ (10 marks)
Test the convergence of the matrix A using the spectral radius

- c) Define the following terms
- i) Diagonal dominance
 - ii) Positive definite (4marks)