CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTERS OF EDUCATION

MATH 849: MULTIVARIATE ANALYSIS

STREAMS: TIME: 3 HOURS

DAY/DATE: FRIDAY 12/4/2024 11.30 A.M. – 2.30 P.M.

QUESTION 1[20 MARKS]

a) Two samples 1 and 2 were taken were subjected to an experiment where three characteristics X_1, X_2 and X_3were measured. Assume the two samples were taken from $N_3(\mu_1\Sigma_1)$ and $N_3(\mu_2\Sigma_2)$ respectively. The summary statistics are
$n_1 = 16$ and $n_2 = 16$.
$\bar{X'}_1 = [1 \quad 3 \quad 2]$ and $\bar{X'}_2 = [2 \quad 4 \quad 2]$.
The pooled sample variance- covariance matrix is

$$S=\begin{bmatrix}13 & -4 & 3\\ -4 & 13 & -2\\ 3 & -2 & 10\end{bmatrix}.$$

Test the hypothesis at 5% significance level
$H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$. [6 marks]

b) Explain briefly the concept of Principal component analysis. [3 marks]

c) Given that

$$X' = \begin{bmatrix}14 & 12 & 11 & 14 & 16 & 11 & 15\\ 18 & 14 & 16 & 20 & 23 & 21 & 16\end{bmatrix}$$

Use the Hotteling's T^2 statistic to test the hypothesis that the mean vector is not different from μ.

QUESTION 2[20 MARKS]

a) Three vectors $y_1, y_2 and\ y_3$ were observations from $N_3(\mu \quad \Sigma \quad)$ with a mean of $\mu^1 =$ $[4 \quad 1 \quad 3]$ and

$$\Sigma = \begin{bmatrix} 4 & 4 & -2 \\ 4 & 13 & 1 \\ -2 & 1 & 6 \end{bmatrix}.$$

Find the distribution of

i) $Z = y_1 - 2y_2 + 2y_3$

ii) y_2

iii) $y_1\ and\ y_3$

b) The variance-covariance matrix of some three observations is given by

$$S = \begin{bmatrix} 4 & 0 & 0.8 \\ 0 & 9 & 0 \\ 0.8 & 0 & 1 \end{bmatrix}$$

Find the

i. Eigen values [6 marks]

ii. Eigen vector [4 marks]

iii. Normalise the eigen vector

iv. Equation of the first component

v. Total variance explained by the 1st principal component

QUESTION 3[20 MARKS]

a) Given the mean vector and the variance-covariance matrix below,

$\mu^1 = [1 \quad 3 \quad 2]$ and

$$\Sigma = \begin{bmatrix} 5 & -3 & 3 \\ -3 & 7 & -1 \\ 3 & -1 & 10 \end{bmatrix}.$$

Obtain

i. $E[X_1/X_2X_3]$ [6 marks]

ii. $Var[X_1/X_2X_3]$ [2 marks]

b) Explain the concept of discriminant analysis [4 marks]

c) Define the following terms as used in Multivariate analysis

i. Rank [3 marks]

ii. Orthogonal Matrix [1 mark]

d) The joint probability distribution of X_1 *and* X_2 given by

$$f(X_1X_2 = \begin{cases} 24x_1x_{2,}, 0 < x_1 < 1 \; and \; 0 < x_2 < 1 \\ 0, elsewhere \end{cases}$$

Use the expectation method to verify if X_1 *and* X_2 are independent. [4 marks]

QUESTION 4[20 MARKS]

a) Given that $x_i \sim N_n(\underline{\mu}, \Sigma)$, i=1,2,…,n, Find the MLE of μ. [10 marks]

b) The following vectors are the yield of three crops as a result of treatment A and treatment B. Treatment A was replicated five times, while B was replicated three times.

Treatment A

$$\begin{bmatrix} 7 \\ 14 \\ 3 \end{bmatrix}, \begin{bmatrix} 8 \\ 13 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 11 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ 10 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \\ 3 \end{bmatrix}$$

Treatment B

$$\begin{bmatrix} 7 \\ 6 \\ 8 \end{bmatrix}, \begin{bmatrix} 6 \\ 8 \\ 9 \end{bmatrix}, \begin{bmatrix} 4 \\ 10 \\ 8 \end{bmatrix}$$

Obtain

i. The respective mean vectors [1 mark]

ii. The overall mean vector [1 mark]

iii. The matrices of sum of squares due to each treatment. [7 marks]

iv. The combined matrix of sum of squares due to both treatments. [1 mark]

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