

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF SCIENCE IN
CHEMISTRY

CHEM 803: ADVANCED MATHEMATICS FOR CHEMISTS

STREAMS: MSc. CHEM

TIME: 3 HOURS

DAY/DATE: MONDAY 11/08/2025

8.30 A.M. – 11.30 A.M.

INSTRUCTIONS:

- Answer any FOUR Questions.
- Adhere to the instructions on the answer booklet

QUESTION ONE

a. Find the area enclosed between the curve $y = -x^2 + 4$, and $y = x^2 - 2x$ 3 marks

b. Solve the system of equations by the row reduction 3 marks

$$x + 3y - 2z = 1$$

$$4x - 5y + 6z = 12$$

$$3x + 5y + 2z = 19$$

c. If $y = \ln x - \frac{1}{2} \ln(1 + x^2) - \frac{\tan^{-1} x}{x}$, prove that $\frac{dy}{dx} = \frac{\tan^{-1} x}{x^2}$ 5 marks

d. Solve $(x + 1) \frac{dy}{dx} - y = e^x (x + 1)^2$ using a suitable integrating factor 2 marks

e. The half-life of a radioactive substance is 1620 years. If its mass is now 4grams, Find the time when 1.5grams of the substance remain. 2 marks

QUESTION TWO

- a. Evaluate the following limits $\lim_{x \rightarrow \infty} \frac{x^3}{(x+4)(2x^2+1)}$ 2 marks
- b. Given that $y = f(x) = \frac{2}{x^2+3}$, find $\frac{dy}{dx}$ from first principles. 3 marks
- c. Prove that $\frac{dy}{dx}$ of $\cot^{-1}\left(\frac{2}{x}\right) + \tan^{-1}\left(\frac{x}{2}\right) = (\sin^{-1} x)^2$ 5 marks
- d. Find the equations of lines tangent to the curve $y^2 - 6x^2 + 4y = 6$ at the point $y = 0$. 2 marks
- e. Evaluate the following integral

$$\int \frac{dx}{16x^2 + 9}$$
 3 marks

QUESTION THREE

- a. Given that $y = \tan(4x+3)$, find $\frac{dy}{dx}$ 3 marks
- b. Obtain $\frac{d^3y}{dx^3}$ of $x = 4\sec^2 \theta$, and $y = 4\tan^3 \theta$ 5 marks
- c. Verify whether the differential equation below is exact hence Solve $(4x^3y^3 + 3x^2)dx + (3x^4y^2 + 6y^2)dy = 0$ 2 marks
- d. Obtain $\frac{d^2y}{dx^2}$ of $y = e^{xy}$ 5 marks

QUESTION FOUR

- a. Given the function defined as $f(x) = \begin{cases} x-1 & \text{if } x > 0 \\ x^2-1 & \text{if } x \leq 0 \end{cases}$ Sketch the graph of the function and obtain $f(0)$ and $f(2)$ 2 marks
- b. Find the general solution of $y'' + 6y' + 5y = e^{2x}$ 3 marks
- c. Obtain $\frac{d^2y}{dx^2}$ of $4y^3 - x^2 = 10$ 5 marks

d. Evaluate $\int_0^1 5x^2 e^{4x} dx$ using the by parts method 3 marks

e. Solve the simultaneous equations below both by the Cramers rule method

$$x + y + z = 10$$

$$2x - 3y + 5z + 9 = 0$$

$$x + 2y - z = 6$$

2 marks

QUESTION FIVE

a. Evaluate the angle between the two vectors,

$$a = i - 2j + 4k \quad \text{and} \quad b = -4i + j - 2k$$

3 marks

b. Find the area of the triangle PQR with vertices $P=(3 \ 4 \ 7)$, $Q=(0 \ 6 \ 1)$ and $R(5 \ -2 \ 4)$

3 marks

c. State the order, linearity and degree of the following differential equation giving reasons

$$\left(1 + \left(\frac{d^4 y}{dx^4}\right)^2\right)^3 = \left(\frac{d^3 y}{dx^3}\right)^3 + e^{2x}$$

3 marks

d. Determine whether $y = \frac{x^2}{3} + \frac{1}{x}$ is a solution of the differential equation $x \frac{dy}{dx} + y = x^2$

3 marks

e. Given that $y \cos x = e^x$, evaluate $\frac{d^2 y}{dx^2} - 2 \tan x \frac{dy}{dx} - 2y$ 3 marks

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