CHUKA **UNIVERSITY**

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF AGRICULTURAL ECONOMICS

AGEC 854: MATHEMATICS FOR ECONOMISTS

STREAMS: MSC. AGEC **TIME: 3 HOURS**

DAY/DATE: MONDAY 14/04/2025 **2.30 P.M. – 5.30 P.M.**

INSTRUCTIONS

- **Answer question ONE and any other TWO**
- **Show all calculations for mathematical problems.**

QUESTION ONE

a) Find the first derivative of the function $f(x) = x^2 \ln(x), x > 0$ **(5 Marks)**

b) Using the integration by parts, evaluate the definite integral: **(5 Marks)**

$$\int_1^3 x\, ln(x)\, dx$$

c) A farmer wants to maximize profit given by:

$$\pi(x, y) = 8x + 6y$$

subject to a budget constraint for inputs:

$$3x + 2y = 100$$

where *x* and *y* are the quantities of two different inputs.

i. Set up the Lagrangian function. **(2 Marks)**

ii. Derive the first-order conditions. **(3 Marks)**

iii. Solve for the optimal values of *x* and *y* and compute the maximum profit. **(5 Marks)**

QUESTION TWO

A farm uses a crop rotation system among three crops: maize (M), wheat (W), and soybeans (S).

The annual crop rotation probabilities are given by the following transition matrix:

$$P = \begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & 0.4 & 0.2 \\ 0.3 & 0.2 & 0.5 \end{pmatrix}$$

The current year's planting distribution is represented by the state vector:

$$S_0 = (0.4 \quad 0.35 \quad 0.25)$$

a) Compute the state vector for next year. **(7 Marks)**

b) Compute the state vector for two years from now. **(7 Marks)**

c) Determine the steady-state distribution . **(6 Marks)**

QUESTION THREE

i) A cost function for producing crop *x* is given by:

$$C(x) = 5x^3 - 30x^2 + 60x + 100$$

a) Find the first derivative C′(*x*) and interpret its economic meaning (marginal cost).

(3 Marks)

b) Determine the level of production *x* that minimizes the cost by finding the critical points and using the second derivative test. **(5 Marks)**

ii) Use the simplex to solve the system of linear equations: **(12 Marks)**

$$2x + y - z = 5$$

$$x - 3y + 2z = -4$$

$$3x + y + z = 6$$

QUESTION FOUR

i) Using Kuhn-Tuckers conditions to solve the following maximization problem

$Max\ U = xy^2$

Subject to

$x + y \leq 100$

$2x + y \leq 120$

$x, y \geq 0$ **(8 Marks)**

ii) In a village, farmers grow maize (M), beans (B), and potatoes (P). The data shows:

- 100 farmers grow maize,
- 80 farmers grow beans,
- 60 farmers grow potatoes,
- 40 grow both maize and beans,
- 30 grow both maize and potatoes,
- 20 grow both beans and potatoes,
- 10 grow all three crops.

a) Represent this information in a Venn diagram. **(4 Marks)**

b) How many farmers grow only maize? **(2 Marks)**

c) How many farmers grow all three crops? **(2 Marks)**

d) How many farmers grow at least two crops? **(2 Marks)**

e) How many farmers grow none of these crops, assuming there are 200 farmers in total? **(2 Marks)**

……