CHUKA UNIVERSITY

Knowledge is wealth (*Sapientia divitia est*)

UNIVERSITY EXAMINATIONS
FIRST YEAR EXAMINATION FOR THE AWARD OF MASTERS OF SCIENCE IN APPLIED MATHEMATICS

MATH 826: NUMERICAL ANALYSIS II **TIME: 3 HOURS**
DAY/DATE:

INSTRUCTIONS:
Answer Any Three Questions

QUESTION ONE (20 MARKS)

a) Use Picard method to find the value of y when x=0.1 and y=1 when x=0 and $\frac{dy}{dx} = 3x + y^2$ Perform five iterations. (10 marks)

b) Apply Euler method in finding the solution of $\frac{dy}{dx} = x + y;\ y(0) = 1$. Using h=0.2 and $0 < x < 1$, determine the error and the percentage error at each step given that the exact solution is $y = -1 - x + 2e^x$ (10marks)

QUESTION TWO (20 MARKS)

a) Use the 2nd order Runge-Kutta method to integrate $\frac{dy}{dx} = \sin y$ with y(0)=1, $0 \le x \le 0.5$ and h=0.1. Give your answer to 4 decimal places (8 marks)

b) Solve the differential equation using 4th order RungeKutta method (8 marks)
$y' = x - y$ given that $x_0 = 0$, $y_0 = 2$ and h=0.2

c) Write the main principle of predictor corrector methods in solving ordinary differential equation (4 marks)

QUESTION THREE (20 MARKS)

a) Use Adams Bashforth Morton method with h=0.2 to obtain approximation to y(0.8) for the solution of the differential equation (10marks)
$\frac{dy}{dx} = x + y - 1$; y(0)=1

b) Given that $\frac{dy}{dx} = \frac{1}{2}\left(1 + x^2\right)y^2$ and y(0)=1, y(0.1)=1.06, y(0.2)=1.12 and y(0.3)=1.21. Evaluate y(0.4) using Milner predictor corrector method (10marks)

QUESTION FOUR (20 MARKS)

a) Use Euler method to obtain the value y(0.2), where y(x) is the solution of the initial value problem $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ with y(0)=1 , $y'(0) = 2$ (10 marks)

b) Find the error bound in using the composite Trapezoid rule on the interval $\int_0^4 \cos(3x)dx$ from using 200 subinterval of width h (10 marks)

c) Define the following terms (4 marks)
   i) Initial value problem
   ii) Boundary value problem

QUESTION FIVE (20 MARKS)

a) Find the upper bound for the error in estimating $\int_0^x 5x^4 dx$ using Simpson's rule with n=4 (3marks)

b) Use Runge Kutta 4th order method to approximate x at 0.6 and y at 0.6 with h=0.2 given the initial value problem
   $x' = 2x + 4y$
   $y' = -x + 6y$
   X(0)=-1 and y(0)=6 (12marks)

c) Find the degree of precision for the Trapezoid rule. Focusing on only the interval $[-1, 1]$ (5marks)