CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN ACTURIAL SCIENCE

ACMT 212: PRINCIPLES OF OPERATIONS RESEARCH

STREAMS: **TIME: 2 HOURS**

DAY/DATE: WEDNESDAY 18/12/2024 **11.30 A.M – 1.30 P.M**

INSTRUCTIONS:

Question One

a) Define Integer Programming. (2 marks)

b) Define what is ***service time.*** (2 marks)

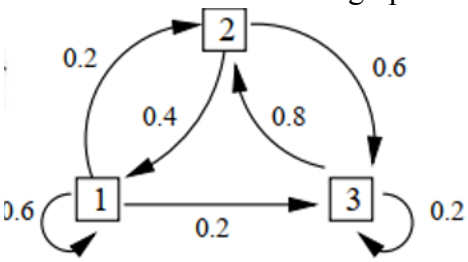
c) Differentiate between a recurrent and an absorbing state in Markov chains. (4 marks)

d) Compare the Newton-Raphson method with the bisection method in terms of convergence speed and reliability. (4 marks)

e) Describe three applications of random numbers in real-world scenarios. (6 marks)

f) Formulate the following problem as a linear programming model: A factory produces two units of material available. Each unit of Product 1 requires 2 hours of labor and 3 units of material. Each unit of Product 2 requires 1 hour of labor and 2 units of material. The profit from Product 1 is Kshs.30, and from Product 2 is Kshs.20. Maximize the profit.(4 marks)

g) Given the below transition graph



i)Find the transition matrix p? (3 marks)

ii)Find $P(x_2=1 \mid x_0=1)$. (5 marks)

Question two

a) What is sensitivity analysis?What happens to your existing solution when we increase the amount of resource in a non-binding constraint? (4 marks)

b) State the Markov Property .(2 marks)

c) Outline the phases of an Operations Research study. (5 marks)

d) You are a project manager deciding between two potential projects, P1 and P2. Each project has uncertain outcomes, and you have estimated the following payoff table in thousands of dollars:

Project	High Return	Medium Return	Low Return
P1	Kshs.20	Kshs.15	Kshs.5
P2	Kshs.18	Kshs.17	Kshs.10

i. Determine the best project using the maximin criterion. (3 marks)

ii. Determine the best project using the maximax criterion. (3 marks)

iii. Determine the best project using the minimax regret criterion. (3 marks)

Question three

a) In a discrete-time Markov chain, there are two states 0 and 1. When the system is in state 0 it stays in that state with probability 0.4. When the system is in state 1 it transitions to state 0 with probability 0.8 . Graph the Markov chain and find the state transition matrix P. (4 marks)

b) Given the primal problem: Maximize $z = 3x_1 + 2x_2$ subject to $x_1 + x_2 \leq 4$,$2x_1 + x_2 \leq 6$,x_1, $x_2 \geq 0$,

i. Formulate the dual problem. (4 marks).

ii. Provide an economic interpretation of the dual variables. (4 marks)

c) A small retail store sells a popular gadget. The daily demand for this gadget is modeled as a discrete random variable with the following probability distribution:

- Demand (units): 0, 1, 2, 3, 4, 5
- Probability: 0.10, 0.20, 0.30, 0.20, 0.15, 0.05

Simulate the daily demand for 10 days. Use the following uniform random numbers for the simulation: 0.72, 0.09, 0.20, 0.44, 0.52, 0.83, 0.25, 0.59, 0.94, 0.17. (8 marks)

Question Four

a) For the function $f(x)=x^3-x-2$, find the derivative $f'(x)$. Use the Newton-Raphson method with an initial guess of $x_0=1.5$ to find an approximate root. Perform two iterations. (6 marks)

b) Consider a construction project with the following tasks and dependencies:

Task	Duration(days)	Predecessor
A	3	None
B	5	A
C	2	A
D	4	B
E	6	C,D
F	3	C
G	5	E,F

i. Construct the project network diagram **showing all tasks, their durations, and dependencies.(5 marks)**

ii. Calculate the early start (ES), early finish (EF), late start (LS), and late finish (LF) times **for** each task. Identify the critical path and determine the project duration. (9 marks)

Question Five

a) A firm is evaluating whether to launch a new product. Given the below projected information, advise whether the firm should launch the product? (5 marks)

Launch the product (Cost: Kshs.50,000)

- Success (Probability: 0.6, Profit: Kshs.200,000)
- Failure (Probability: 0.4, Loss: Kshs.50,000)

Do not launch (No cost or profit)

b) Use the simplex method to solve the LP problem: Maximize $z = 3x_1 + 2x_2$

subject to $x_1 + x_2 \leq 4$, $x_1 \leq 2$, $x_2 \leq 3$, $x_1, x_2 \geq 0$ (15 marks)
