EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF ACTUARIAL SCIENCE STREAMS: TIME: 2 HOURS

MATH 453: FUNDAMENTALS OF BAYESIAN INFERENCE AND DECISION THEORY

DAY/DATE:

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

(a) Define the following terms
  i. zero-sum two-person game (2 Marks) ii.
  saddle point (2 Marks)

(b) The table below gives the payoff matrix for a 2 by 2 zero-sum two-person game:

	Player A	
Player B	9	-2
	0	6

Find the randomized strategies that should be adopted by each player. (4 Marks)

(c) The punctuality of trains has been investigated by considering a number of train journeys. In the sample, 60% of trains had a destination of Manchester, 20% Edinburgh and 20% Birmingham. The probabilities of a train arriving late in Manchester, Edinburgh or Birmingham are 30%, 20% and 25%, respectively. A late train is picked at random from the group under consideration. Calculate the probability that it terminated in Birmingham. (6 Marks)

(d) Consider a game given in the matrix below. Using the dorminance strategy, reduce the game as low as possible. (4 Marks)

		Player A			
		I	II	III	IV
	1	2	0	1	4
Player B	2	1	2	2	3
	3	4	1	3	2

(e) Calculate a 95% confidence interval for the average height of 10-year-old children, assuming that heights have a distribution (where and are unknown), based on a random sample of 5 children whose heights are: 124cm, 122cm, 130cm, 125cm and 132cm. (5 Marks)

(f) Suppose that in a case-control study, we trace 51 smokers in a group of 83 cases of lung-cancer and 23 smokers in the control group of 70 diseases-free subjects. The preference rate (estimates of the proportion of the disease at the population) of lungcancer is equal to 1%. Calculate the probability that a smoker will develop lung-cancer. (4 Marks)

(g) A specialist insurer that provides insurance against breakdown of photocopying equipment calculates its premiums using a credibility formula. Based on the company's recent experience of all models of copiers, the premium for this year should be Ksh. 250 per machine. The company's experience for a new model of copier, which is considered to be more reliable, indicates that the premium should be Ksh. 90 per machine. Given that the credibility factor is 0.76, calculate the premium that should be charged for insuring the new model. (3 Marks)

QUESTION TWO (20 MARKS)

(a) Differentiate between a prior distribution and a posterior distribution. (4 Marks) (b) The annual number of claims from a particular risk has a Poisson distribution with mean . The prior distribution for has a gamma distribution with and .

Claim numbers over the last *n* years have been recorded.

i. Show that the posterior distribution is gamma and determine its parameters. (4 Marks) ii. Given that and determine the Bayesian estimate for under:

a. squared-error loss (2 Marks)
b. all-or-nothing loss (6 Marks)
c. absolute error loss (4 Marks)

QUESTION THREE (20 MARKS)

(a) An experiment was carried out to find out the number of hours that actuarial students spend watching television per week. It was discovered that for a sample of 10 students, the following times were spent watching television:

6, 4, 8, 5, 9, 7, 3, 9, 5, 7

Calculate a symmetrical 95% confidence interval for the mean time an actuarial student spends watching television per week. Write down the assumptions needed to find this confidence interval. (6 Marks)

(b) A general insurance company is debating introducing a new screening programme to reduce the claim amounts that it needs to pay out. The programme consists of a much more detailed application form that takes longer for the new client department to process. The screening is applied to a test group of clients as a trial whilst other clients continue to fill in the old application form. It can be assumed that claim payments follow a normal distribution.

The claim payments data for samples of the two groups of clients are:

Without screening	24.5	21.7	35.2	15.9	23.7	34.2	29.3	21.1	23.5	28.3

With screening	22.4	21.2	36.3	15.7	21.5	7.3	12.8	21.2	23.9	18.4

i. (a) Calculate a 95% confidence interval for the difference between the mean claim amounts. (7 Marks)
(b) Comment on your answer. (2 Marks)

ii. (a) Calculate a 95% confidence interval for the ratio of the population variances. (3 Marks)
(b) Hence, comment on the assumption of equal variances required in part (i). (2 Marks)

QUESTION FOUR (20 MARKS)

(a) A consumer has to decide whether to take out a travel insurance policy which covers him for all trips over the next year, or to purchase a separate policy each time he makes a trip. He is unsure how many trips he will make. An annual policy would cost £95. A separate policy for each trip costs £30.

He estimates that the probability distribution of the number of trips he will take over the next year, X, is as follows:

Number of trips, x P(X=x)
10.1 2 0.2 3 0.3 4 0.3
5 0.1

Determine the minimax and Bayes decisions. (6 Marks)

(b) A market trader has the option for one day of selling either ice-cream , hot food or umbrellas at an outdoor festival. He believes that the weather is equally likely to be fine , overcast or wet and estimates his profits under each possible scenario to be:

	25	19	7
	10	30	8
	0	2	34

i. Determine the minimax solution to this problem (2 Marks)

ii. The trader's partner is very optimistic and believes that the criterion to adopt in deciding which product to sell should be to maximize the maximum profit. Determine the decision the trader's partner would make based on these predicted profits. (2 Mark)

iii. Determine the Bayes criterion solution to this problem. (2 Marks) iv. The trader's partner agrees that it is equally likely to be either fine or wet but believes that there is more than an evens chance of it being overcast. By sketching a graph of the Bayes risk for each of the three possible decisions against the probability of it being overcast (p), or otherwise, determine the revised Bayes criterion solution. (8 Marks)

QUESTION FIVE (20 MARKS)

(a) An insurer is setting the premium rate for the buildings in an industrial estate. Past experience for the estate indicates that a premium rate of £3 per £1,000 sum insured should be charged. The past experience of other similar estates for which the insurer provides cover indicates a premium rate of £5 per £1,000 sum insured. If the insurer uses a credibility factor of 75% for this risk, calculate the premium rate per £1,000 sum insured. (2 Marks)

(b) Let denote the proportion of insurance policies in a certain portfolio on which a claim is made. Prior beliefs about are described by a beta distribution with parameters and . Underwriters are able to estimate the mean, and variance, of . i. Express and in terms of and . (6 Marks)

A random sample of n policies is taken and it is observed that claims had arisen on d of them.

ii. (a) Determine the posterior distribution of (3 Marks)
(b) Show that the mean of the posterior distribution can be written in the form of a credibility estimate. (4 Marks)

iii. Show that the credibility factor increases as increases. (3 Marks) iv. Comment on the result in part (iii). (2 Marks)