

**ON BROWDER SPECTRUM AND QUASISIMILARITY OF SOME
CLASSES OF OPERATORS IN HILBERT SPACES**

FAITH WAIRIMU KAMAU

**A Thesis Submitted to the Graduate School in Partial Fulfillment of the
Requirements for the Award of the Degree of Master of Science in Pure
Mathematics of Chuka University**

CHUKA UNIVERSITY

OCTOBER, 2024

DECLARATION AND RECOMMENDATION


Declaration


This thesis is my original work and has not been presented for an award of a degree in any other university.

Signature:  Date: 22/10/2024
Faith Wairimu Kamau
SM13/51169/21

Recommendation

This thesis has been examined, passed and submitted with our approval as university supervisors

Signature:  Date: 22/10/2024
Prof. Sammy Musundi Wabomba, Ph.D.
Chuka University.

Signature:  Date: 22/10/2024
Dr. Jeremiah Ndung'u Kinyanjui, Ph.D.
Kirinyaga University.



COPYRIGHT

©2024

All rights reserved. No part of this research project may be reproduced or transmitted in any form or by any means of mechanical photocopying, recording or any information storage or retrieval system, without prior permission in writing from the author or from Chuka University.

DEDICATION

I dedicate this work to God for His grace and guidance, to my parents for their steadfast support that has been a constant source of strength and to my brother Francis Ngigi Kamau for his encouragement and belief in me which have been a driving force throughout this journey.

ACKNOWLEDGEMENT

I am grateful to Almighty God for His providence and guidance throughout my research period. I would like to express my deepest and sincere gratitude to my supervisors Prof. Musundi Sammy and Dr. Kinyanjui Jeremiah for their invaluable guidance, support, encouragement and mentorship in my research work. Their patience, expertise, insightful feedback, constructive criticism, mentorship and unwavering commitment have shaped the direction and quality of this research.

My sincere gratitude to my parents and siblings for their love, support, encouragement and believing in my abilities which has been a great motivation and inspiration during my research.

Also, a special gratitude to Pauline and Lilian for their moral support that have been instrumental in overcoming various challenges encountered during the research process. I am indebted to my friends and colleagues for their endless encouragement and assistance that enabled me never to give up on my research work.

Finally, I would like to thank Chuka University for providing the necessary resources and facilities that enabled me to conduct my research effectively.

ABSTRACT

The spectrum of operators and quasisimilarity on classes of operators in Hilbert Spaces has been extensively studied. It has been shown that quasisimilarity on m -hyponormal, w hyponormal and (p,k) quasihyponormal operators have equal spectrum, equal Essential Spectrum and equal Weyl Spectrum. However, the consideration of equality of the Browder spectrum for such operators together with quasisimilarity has not been done. Therefore, this study considered the properties of Browder spectrum and quasisimilarity on classes of operators such as m -hyponormal, w -hyponormal and (p,k) quasihyponormal operators and established that quasisimilarity preserves normality, hyponormality, m -hyponormality, w -hyponormality, log-hyponormality and (p,k) -quasihyponormality. We also determined the conditions under which quasisimilarity of such operators yields equal Browder spectrum, that, the operators must be quasisimilar and intersected with biquasitriangular operators. In addition, the study determined the relationship of Browder spectrum and Numerical range and established the spectral theory of Browder spectrum. Finally, the study established an equivalent relation of Browder spectrum and Single-valued extension property. In achieving this, the properties of normal operators, hyponormal operators, biquasitriangular and spectral properties of classes of operators together with properties of Single-Valued Extension Property (SVEP) and Browder's theorem were considered. The results that have been established on other equivalence relation on these classes of operators on almost similarity and unitary equivalence were used in comparison. The findings on the Browder spectrum and quasisimilarity of these classes of operators will be useful in other branches of Mathematics and Physics such as differential operators and algebra. The study of the spectrum of operators will be important for ones applying these operators in spectral analysis, algebra, fluid mechanics, quantum mechanics and differential operators for the calculations of wave function and formulation of theory.