

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN
ACTUARIAL SCIENCE

ACMT 301: LIFE CONTINGENCIES 1

STREAMS: BSC

TIME:2 HOURS

DAY/DATE: TUESDAY 17/12/2024

11.30 A.M. –1.30 P.M.

Question one (30 marks)

A. Consider an insurance policy which pays kshs 100,000 if an individual dies during a year.

Let X = pay out. Suppose $P(\text{individual dies})=0.01$ then

$$P(x=\text{kshs } 100,000) = 0.01$$

$$P(X=0) = 0.99$$

Find $E(X)$ **(4 marks)**

B. List five properties of a survival function.

(4 marks)

C. Explain five factors that affect observed mortality rates. Give examples. (5 marks)

D. A life office has just sold a 25 year term assurance policy to a life aged 40. The sum assured is kshs 50,000 and is payable at the end of the year of death. Calculate the variance of the present value of this benefit. Assume AM92 ultimate mortality and 4% p.a interest

(5 marks)

E. $S(x) = \frac{1}{10} \sqrt{100 - X} \quad 0 \leq x \leq 100$

i) Find the probability that a life aged 19 will die before age 36.

(2 marks)

ii) Find the probability that a life aged 19 dies between ages 36 and 75.

(3 marks)

F. Define a life table.

(2 marks)

G. A life aged exactly 33 years purchases a whole life policy with a sum assured of kshs 40,000 payable at the end of the year of death premiums of ksh 520 are payable in advance. Calculate the variance of the insurer's profit on this contract assuming AM92 ultimate mortality and 4% p.a interest

(5 marks)

Question two (20 marks)

- A. Out of 100 lives aged x , 4 die at age $X+1/4$, TWO $x+1/2$ and FOUR $x+3/4$. Find
- i) Total years lived between $(x, x+1)$ **(2 marks)**
 - ii) Central death rate at age x **(2 marks)**
 - iii) Average years lived between $(x, x+1)$. **(2 marks)**
- B. In a certain population the force of mortality equals to 0.025 for all ages. Calculate
- i) The probability that a new born baby will survive to age 5.
 - ii) The probability that a life aged 10 will die before age 12.
 - iii) The probability that a life aged 5 will die before 10 and 12.
 - iv) The complete expectation of a life of a new born baby.
 - v) The curtate expectations of a life of a new born baby.
- (10 marks)**
- C. Define
- i) Random Experiment
 - ii) Sample space **(4 marks)**

Question three (20 marks)

- A. Given the survival function
- $$S(x) = e^{-0.05x} \quad x > 0$$
- i) Calculate $5/10 \text{ } q_{30}$ **(3 marks)**
 - ii) $F(30)$ **(3 marks)**
 - iii) Calculate e^0_{30} **(3 marks)**
 - iv) Calculate variance $[T(x)]$ **(4 marks)**
- B. If $lx = 1000 - 10x$ for $90 \leq x \leq 100$ write down an expression for $a^{(4)}_{90}$ **(4 marks)**
- C. A male pension policyholder is aged 50 and he will retire at age 65, from which age a pension of kshs 5000 p.a will be paid annually in advance. Before retirement he is assumed to experience mortality in line with PMA92C20. Calculate the expected present value of the benefits assuming interest of 4% p.a. **(3 marks)**

Question 4 (20 marks)

- A. Consider the following mortality pattern of a cohort of 21 lives aged 90
- $$\cdot_{90}=6, \cdot_{91}=, \cdot_{92}=3, \cdot_{93}= \cdot_{94}= \cdot_{95}= \cdot_{96}=2, \cdot_{97}=1$$

Calculate $e_x = E(K)$ AND $\text{Var}(k)$ **(10 marks)**

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B. There are 100 persons now age 40 of whom 19 are expected to die before age 41.
Determine ${}_{0.5}q_{40+0.375}$ assuming

- i) UDD (3 marks)
- ii) Balducci (3 marks)
- iii) Constant force of mortality. (4 marks)

Question Five (20 marks)

- A. Deduce the de Moivre law of mortality. (4 marks)
 - B. Express by a single symbol the probability that a life aged 50, insured three years ago, will die in his 59th year between ages 58 and 59, assuming a five year select period. (5 marks)
 - C. Premiums are paid to the insurer. Explain three ways in which premiums are paid. (6 marks)
 - D. Calculate the annual premium for a term assurance with a term of 10 years to a male aged 30, with a sum assured of kshs 500,000, assuming AM92 ultimate mortality and interest of 4% p.a. Assume that the death benefit is paid at the end of the year of death. (5 marks)
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