

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN
MATHEMATICS

MATH 427: PARTIAL DIFFERENTIAL EQUATIONS II

STREAMS: BSC

TIME: 2 HOURS

DAY/DATE: FRIDAY 01/09/2023

8.30 A.M – 10.30 A.M.

INSTRUCTIONS

- Answer ALL questions
- Adhere to the instructions on the answer booklet.

QUESTION ONE

- a) Classify the following differential equation in the second quadrant of the xy plane.

$$\frac{1}{2}(\sqrt{y^2 + x^2})U_{xx} + 2(x - y)U_{xy} + \frac{1}{2}(\sqrt{y^2 + x^2})U_{yy} = 0 \quad 4 \text{ marks}$$

- b) Verify that the function $U = ae^{-\left(\frac{x}{h} - \frac{ct}{h^2}\right)^2}$ satisfies the wave equation $\frac{1}{c^2} \frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 U}{\partial x^2}$ 4 marks

- c) Solve the pde $\frac{\partial^2 U}{\partial x \partial y} = e^y \cos x$ given that $U(0, y) = -e^y \sin x$ by direct integration 5mks

- d) Given the pde $U_{xx} - 2U_{xy} + U_{yy} = \sin x$, evaluate $U(x, y)$ by the D operator 6 marks

- e) Apply the method of separation of variables to solve $U_{xx} = U_t + U$, given that

$$U(x, 0) = x^2(25 - x^2) \quad 6 \text{ marks}$$

- f) Given the partial differential equation $U_{xx} + 4U_{xy} + 5U_{yy} = 0$

- i. Find the characteristics ε and η 4 marks
- ii. Obtain U_{xx} and U_{yx} in terms of the characteristics 4 marks

QUESTION TWO

- a. Solve for $U(x, y)$ given the pde $U_{xy} = x^2y$, Boundary conditions $U(x, 0) = x^2$ and $U(1, y) = \cos y$ 7 marks
- b. Solve the Laplace equation $U_{rr} + \frac{1}{r}U_r + \frac{1}{r^2}U_{\theta\theta} = 0$ by separation of variables given the boundary conditions
 - (i) $u(r, 0) = 0$, (ii) $u(r, \pi) = 0$, (iii) $u(a, 0) = T$ 7 marks
- c. Given that the homogeneous PDE $A(x, y)U_{xx} + 2B(x, y)U_{xy} + C(x, y)U_{yy} = 0$ is parabolic, Find it's characteristic curve λ , and show that $\lambda = \frac{-B}{C}$. 6 marks

QUESTION THREE

- a. Given the Pde $U_{xx} + U_{xy} - 6U_{yy} = y \cos x$, Obtain the solution of the Pde by the D operator 7 marks
- b. Obtain the solution to the non homogeneous Pde $U_{xx} + U_x - U_{yy} + 3U_y - 2U = x^2y$ 7 marks
- c. Given the PDE, $(3 + y)U_{xx} + 2(3 - x)U_{xy} + (3 + y)U_{yy} = U_x + U_y$, determine the values of (x) and (y) for which the equation is
 - i. Hyperbolic. 2 marks
 - ii. Parabolic 2 marks
 - iii. Elliptic 2 marks