CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS: 2023

FIRST YEAR EXAMINATIONS FOR THE AWARD OF BACHELOR OF SCIENCE IN MATHEMATICS.

MATH 427: PARTIAL DIFFERENTIAL EQUATIONS II TIME: 2 HOURS

INSTRUCTIONS

Answer question one and any other two questions

Adhere to the instructions on the answer booklet.

QUESTION ONE Compulsory.

a) Classify the following differential equation in the second quadrant of the xy plane.
$\frac{1}{2}\left(\sqrt{y^2+x^2}\right)U_{xx}+2(x-y)U_{xy}+\frac{1}{2}\left(\sqrt{y^2+x^2}\right)U_{yy}=0$ 4mks

b) Verify that the function $U=ae^{-\left(\frac{x}{h}-\frac{ct}{h2}\right)2}$ satisfies the wave equation $\frac{1}{c^2}\frac{\partial^2U}{\partial t^2}=\frac{\partial^2U}{\partial x^2}$ 4mks

c) Solve the pde $\frac{\partial^2U}{\partial x\partial y}=e^y\cos x$ given that $U(0,y)=-e^y\sin x$ by direct integration 5mks

d) Given the pde $U_{xx}-2U_{xy}+U_{yy}=\sin x$, evaluate U(x, y) by the D operator 6mks

e) Apply the method of separation of variables to solve $U_{xx}=U_t+U$, given that
$U(x,0)=x^2\left(25-x^2\right)$ 6mks

f) Given the partial differential equation $U_{xx}+4U_{xy}+5U_{yy}=0$

i. Find the characteristics ε *and* η 4mks

ii. Obtain U_{xx} and U_{yx} in terms of the characteristics 4mks

QUESTION TWO

a. Obtain the solution to the pde $U_{xx}-U_{xy}=\sin x\cos 2y$ by the D operator 7mks

b. The vibrations of an elastic string is governed by the equation $U_{xx} = U_{tt}$. Find the deflection U(x,t) of the vibrating string for t > 0, under the following conditions. 10mks

$U(0,t) = 0$

$U(\pi,t) = 0$

$U_t(x,0) = 0$

$U(x,0) = 2(\sin x + \sin 3x)$

c. Classify the following PDE's

(i). $U_{tt} = 4U_{xx}$ 2mks

(ii). $2U_t = 3U_{xx}$ 2mks

(iii). $U_{xx} - 4U_{xy} + U_{yy} = 0$ 2mks

QUESTION THREE

a. Solve for U(x, y) given the pde $U_{xy} = x^2 y$, Boundary conditions $U(x,0) = x^2$ and $U(1,y) = \cos y$ 6mks

b. Solve the Laplace equation $U_{rr} + \frac{1}{r}U_r + \frac{1}{r^2}U_{\theta\theta} = 0$ by separation of variables given the boundary conditions

$(i)\ u(r,0) = 0,\ (ii)\ u(r,\pi) = 0,\ (iii)\ u(a,0) = T$

c. Given that the homogeneous PDE $A(x,y)U_{xx} + 2B(x,y)U_{xy} + C(x,y)U_{yy} = 0$ is parabolic, Find it's characteristic curve λ, and show that $\lambda = \frac{-B}{C}$. 4mks

QUESTION FOUR

a. Given the Pde $U_{xx} + U_{xy} - 6U_{yy} = y\cos x$, Obtain the solution of the Pde by the D operator 7mks

b. Obtain the solution to the non homogeneous Pde $U_{xx} + U_x - U_{yy} + 3U_y - 2U = x^2 y$ 7mks

c. Given the PDE, $(3+y)U_{xx} + 2(3-x)U_{xy} + (3+y)U_{yy} = U_x + U_y$, determine the values of (x) and (y) for which the equation is

i. Hyperbolic. 2mks

ii. Parabolic 2mks

QUESTION FIVE

a. Show that the function $\Psi = uy\left(1-\dfrac{a^2}{x^2+y^2}\right)$ satisfies the laplace equation 4mks

b. Solve the pde $U_{xx} + U_{yy} = x^2y^2$ by the D operator 8mks

c. Given the Pde $U_{xt} = e^{-t}\cos x$, apply the method of separation of variables to solve the Pde given that $U(x,0) = 0$ and $U_t(0,t) = 0$ 8mks