

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE IN BACHELOR OF SCIENCE
IN ACTUARIAL SCIENCE

ACMT 202: FUNDAMENTAL OF ACTUARIAL MATHEMATICS II

STREAMS:

TIME: 2 HOURS

DAY/DATE: TUESDAY 18/4/2023

2.30 P.M. – 4.30 P.M.

Question One (30 Marks)

- a) Define a contingent payment model (2 marks)
- b) Consider a policy which consists of a payment of Kshs. 50,000 contingents upon retirement in 15 years. Suppose that the probability of a 45 years old to retire in 15 is 0.82 i.e. ${}_{15}P_{45} = 0.82$. Find the actuarial present value of the contingent payment. Assume, a 6% interest compounded annually. (5marks)
- c) A whole life insurance is used to (20) is payable at the moment of death. You are given (1) the time at death random variable is exponential with $\delta = 0.02$ (ii) $\delta = 0.14$. Calculate (i) $\bar{A}_{20}$ (ii) ${}^2\bar{A}_{20}$ and $\text{Var}(\bar{Z}_x)$ (8marks)
- d) Define an n-year term life insurance. Deduce its present random variable, its actuarial present value and its variance. (8 marks)
- e) The age of death random variable is exponential with parameter 0.5. A life aged 35 buys 25year life insurance policy that pays I upon death. Assume an annual effective interest rate of 7%. Find the actuarial present value of this policy (5 marks)

- f) What is an endowment policy? Give two types of endowment policies. (2 marks)

Question Two (20 marks)

- a) Define a term life insurance and state its,
 i). Present value random variable.
 ii). The actuarial present value.
 iii). Its second moments
 iv). Its variance (10 marks)
- b) Let the remaining lifetime at birth random variable X be uniform on $(0,100)$. Let ${}_{101}Z_{30}$ be a contingent payment random variable for a life aged $x=30$.

Find i). ${}_{101}A_{30}$

ii). ${}_{101}^2A_{30}$

$\text{Var} ({}_{101}Z_{30})$ if $\delta = 0.05$ (10 marks)

Question 3 (20 marks)

- a) Explain what recursion relations for life insurance are. Give two types of recursion relations (4 marks)
- b) Show that $A_{x:n}^1 = vq_x + vP_x A_x + 1_{n-1}$ (5marks)
- c) Define an annually increasing whole life insurance (2 marks)
- d) Give that $K(x)$ is uniform on $(0,3)$. Calculate (IA_x) if force of interest $\delta = 0.05$ (5 marks)
- e) Show that $A_{x:n}^1 = 1/\delta A_{x:n}^1 + A_x^1 + n$ (4marks)

Question 4 (20 marks)

- a) Explain what is meant by contingent annuities and give three types of contingent annuities

(8 marks)

b) For a disability insurance claim

- i). The claimant will receive payment at the rate of Kshs. 20,000 per year, payable continuously as long as she remains disabled.
- ii). The length of payment period in years will be a random variable with pdf $f(t) = te^{-t}$
- iii). Payment begin immediately.
- iv). $\delta = 0.05$. Calculate the actuarial present value of the disability payments at the time of disability (6 marks)

c) Explain a continuously n-year temporary life annuity. Find it's the present value random variable and its actuarial present value (6 marks)

Question 5 (20 marks)

- A. What is continuous n-year certain and life annuity. State is present value random variable (4 marks)
- B. You are given the following probability mass function of $k(x)$

K	0	1	2
Pr (kx) = k	0.2	0.3	0.5

Find

$$\ddot{a}_x \text{ and } {}^2\ddot{a}_x = E(\ddot{Y}_x^2) \text{ if } @ i=0.05 \quad (8 \text{ marks})$$

- C. Deduce the actuarial present value of whole life discrete annuity immediate. (4 marks)