ON ORTHOGONALITY AND MICRO TRANSITIVITY CHARACTERIZATION OF HILBERT SPACES

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DECLARATION AND RECOMMENDATION

Declaration

This thesis is my original work and has not been presented for an award of a degree in any other university.

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Recommendation

This thesis has been examined, passed and submitted with our approval as university supervisors.

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DEDICATION

This thesis is sincerely dedicated to my mum, husband and sibling whose unwavering inspiration and support have carried me throughout this research. I also dedicate it to my children Lith and Samuel for their patience, understanding and for offering their full cooperation during this journey.

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ABSTRACT

Characterization of a transitive separable Banach spaces as Hilbert spaces has been an open area of research. It has been shown that separable Banach spaces which are transitive, almost transitive, convex transitive and micro transitive together with isometries of various characteristics such as unitary, reflection, differentiable properties are Hilbert spaces. It has also been shown that a separable real Banach space which is almost transitive with vector orthogonalities of dimension greater than three is a Hilbert space. However, properties such as micro transitive together with vector orthogonalities for n-dimension have essential property that can be utilized to characterize Banach spaces as Hilbert spaces. Additionaly, by this characterization, properties of matrix numerical range and numerical radius can also be determined. Therefore, by utilizing micro transitivity and Isosceles vector (I-vector), Pythagorean vector (P-vector) and Isosceles Pythagorean vector (IP-vector) in the unit sphere of separable Banach space this research determined that an n-dimension separable Banach spaces is a Hilbert space. In addition, by the use of properties of numerical range in general Banach space the study also established properties of matrix numerical range and numerical radius in separable transitive Banach space. The findings of this study will find use in algebra and differential operators for the purpose of calculation of wave functions and formulation of theory. In addition, the findings of the study will find use in spectral analysis of functions for the study of vibrations and interfacial waves stability analysis.