

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF
SCIENCE IN EDUCATIONMATH 453: FUNDAMENTALS OF BAYESIAN INFERENCE AND DECISION
THEORY

STREAMS:

TIME: 2 HOURS

DAY/DATE: FRIDAY 12/4/2024

11.30 A.M. – 1.30 A.M.

INSTRUCTIONS:

Question One

- a) Explain four basic elements in decision theory. (4 marks)
- b) The annual number of claims arising from a particular group of policies follows a Poisson distribution with mean μ . The prior distribution of μ is exponential with mean 30. In the previous two years, the numbers of claims arising from the group were 28 and 26, respectively. Determine the posterior distribution of μ . (8 marks)
- c) The punctuality of trains has been investigated by considering a number of train journeys. In the sample, 60% of trains had a destination of Manchester, 20% Edinburgh and 20% Birmingham. The probabilities of a train arriving late in Manchester, Edinburgh or Birmingham are 30%, 20% and 25%, respectively. A late train is picked at random from the group under consideration. Calculate the probability that it terminated in Manchester. (7 marks)
- d) In a small survey, a random sample of 50 people from a large population is selected. Each person is asked a question to which the answer is either "Yes" or "No." Let the proportion in

the population who would answer “Yes” be θ : Our prior distribution for θ is a beta(1:5; 1:5) distribution. In the survey, 37 people answer “Yes.”

- i) Find the prior mean and prior standard deviation of θ (3 marks)
- ii) Find the likelihood. (2 marks)
- iii) Find the posterior distribution of θ (3 marks)
- iv) Find the posterior mean and posterior standard deviation of θ (3 marks)

Question Two

a) An agricultural company wants to decide which commodity it should stock to get maximum profit. It was supplied with the following information. The probability that the monsoon will be excess, normal and deficient is 0.40, 0.30 and 0.30. The estimated profit or loss of the three commodities in respect of these different kinds of monsoon are:

Profit per 1 ton			
Monsoon	Excess	Normal	Deficient
Rice	10,000	-4,000	15,000
Wheat	4,000	-3,000	8,000
Maize	4,000	1000	-1,000

Determine the optimal decision under each of the following decision criteria and show how you arrived at it:

- I. Maximax (2 marks)
- II. Maximin (2 marks)
- III. Expected monetary value (EMV) (4 marks)

b) Three factories produce light bulbs to supply the market. Factory A produces 20%, 50% of the tools are produced in factories B and 30% in factory C. 2% of the bulbs produced in factory A, 1% of the bulbs produced in factory B and 3% of the bulbs produced in factory C are defective. A bulb is selected at random in the market and found to be defective. What is the probability that this bulb was produced by factory B? (5 marks)

c) An experiment was conducted to compare the strengths of two types of kraft papers, one a standard kraft paper of a specified weight and the other the same standard kraft paper treated with a chemical substance. Ten pieces of each type of paper, randomly selected from production, produced the strength measurements shown in Table below. Use the Mann–Whitney U Test to test the hypothesis of no difference in the distributions of strengths for the two types of paper against the alternative hypothesis that the treated paper tends to be stronger. (7 marks)

Standard i	Treated ii
1.21	1.49
1.43	1.37
1.35	1.67
1.51	1.50
1.39	1.31
1.17	1.29
1.48	1.52
1.42	1.37
1.29	1.44
1.40	1.53

Question Three

a) Discuss in detail any three properties of Bayes estimators. (9 marks)

b) A study is run to evaluate the effectiveness of an exercise program in reducing systolic blood pressure in patients with pre-hypertension (defined as a systolic blood pressure between 120-139 mmHg or a diastolic blood pressure between 80-89 mmHg). A total of 15 patients with pre-hypertension enroll in the study, and their systolic blood pressures are measured. Each patient then participates in an exercise training program where they learn proper techniques and execution of a series of exercises. Patients are instructed to do the exercise program 3 times per week for 6 weeks. After 6 weeks, systolic blood pressures are again measured. The data are shown below. Use the Wilcoxon Signed Rank Test to test whether there is a difference in systolic blood pressures after participating in the exercise program as compared to before? Use level of significance ($\alpha=0.05$). (11 marks)

Patient	Systolic Blood Pressure Before Exercise Program	Systolic Blood Pressure After Exercise Program
1	125	118
2	132	134
3	138	130
4	120	124
5	125	105
6	127	130
7	136	130
8	139	132
9	131	123
10	132	128
11	135	126
12	136	140
13	128	135
14	127	126
15	130	132

Question Four

a) The annual number of claims from a particular risk has a Poisson distribution with mean μ . The prior distribution for μ has a gamma distribution with $\alpha = 2$ and $\lambda = 5$.

Claim numbers x_1, x_2, \dots, x_n over the last n years have been recorded.

- (i) Show that the posterior distribution is gamma and determine its parameters. (7 marks)
- (ii) Given that $n = 8$ and $\sum_{i=1}^8 x_i = 5$ determine the Bayesian estimate for μ under
- (a) squared-error loss (2 marks)
 - (b) all-or-nothing loss (6 marks)
 - (c) absolute error loss. (5 marks)

Question Five

a) It is claimed that the area within which a school is situated affects the age profile of the staff employed at that school. In order to investigate this claim, the age profiles of staff employed at two schools with similar academic achievements are compared.

Academia High School, situated in a rural community, employs 120 staff whilst Best Manor Grammar School, situated in an inner-city community, employs 80 staff.

The percentage of staff within each age group, for each school, is give in the table

Age	Academia High School	Best Manor Grammar School
22-34	17.5	40.0
35-39	60.0	45.0
40-59	22.5	15.0

- I. Form the data into a contingency table suitable for analysis using a χ^2 distribution. (3 marks)
 - II. Use a chi -squared test, at the 1% level of significance, to determine whether there is an association between the age profile of the staff employed and the area within which the school is situated. (15 marks)
 - III. Interpret your result above particularly for the 22-34 age group. (2 marks)
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