

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE, BACHELOR OF EDUCATION AND BACHELOR OF ART

MATH 442: TIME SERIES ANALYSIS

STREAMS: BSc., B.ED, B.A

TIME: 2 HOURS

DAY/DATE: THURSDAY 18/04/2024

11.30 A.M. – 1.30 P.M.

INSTRUCTIONS

- Answer Question ONE and any other TWO Questions
- Unless stated otherwise, assume that $E(e_t) = 0$, $var(e_t) = \sigma_e^2$, $E(e_t X_t) = \sigma^2$, and $Cov(e_t, e_s) = 0$; $s \neq t$
- Standard spectral density function of the White noise is $f_e(\omega) = \frac{\sigma_e^2}{2\pi}$

QUESTION ONE (30 MARKS)

a) Outline **Four** (4) reasons to record and analysis time series data [4 marks]

b) Find the covariance generating functions of

$$X_t = \frac{1}{3}e_{t+1} + \frac{1}{3}e_t + \frac{1}{3}e_{t-1}$$

Hence the autocorrelation function [8 marks]

c) Find the spectral density function of the process X_t given by [8 marks]

$$X_t + X_{t-1} = e_t$$

d) Given the following observation of a time series for $n=10$

t	1	2	3	4	5	6	7	8	9	10
X_t	17	21	30	39	48	60	66	76	89	99

Find

- i. Sample auto-covariance $r(1)$ and $r(2)$ [5 marks]
- ii. Sample auto-correlation $\rho(1)$ and $\rho(2)$ [5 marks]

QUESTION TWO (20 MARKS)

- a) Consider the set independent and identically distributed random variable $\{e_t\}$ such that $E(e_t) = \mathbf{0}$ and variance of e_t is σ_e^2 . Let the process be given by $X_t = \phi e_{t-1} + e_t$ where ϕ is a constant. Show that X_t is weakly stationary. [10 marks]
- b) Consider the AR (2) process given by $X_t = \frac{4}{5}X_{t-1} - \frac{15}{100}X_{t-2} + e_t$. Show that X_t is stationary and find its autocorrelation function. [10 marks]

QUESTION THREE (20 MARKS)

- a) Consider a second order process AR(2) given by $X_t = \frac{1}{3}X_{t-1} + \frac{2}{9}X_{t-2} + e_t$. Show that the ACF of X_t is given by $\rho(k) = \frac{16}{21}\left(\frac{2}{3}\right)^k + \frac{5}{21}\left(\frac{-1}{3}\right)^k, k=0,1,2,3,\dots$ [10 marks]
- b) Fit a local polynomial of degree 2 with 5 consecutive data points given the weight $W = \frac{1}{35}(-3,12,17,12,-3)$ [10 marks]

QUESTION FOUR (20 MARKS)

- a) Outline four (4) application areas where time series data occur naturally. [8 marks]
- b) Outline two (2) approaches to time series analysis [2 marks]
- c) Show that the spectral density function of an AR (1) process $X_t = \theta X_{t-1} + e_t, |\theta| < 1$ is given by

$$f(\omega) = \frac{\sigma^2}{2\pi(1 - 2\theta\cos\omega + \sigma^2)}$$

NB: The standard spectral density function of the white noise is $f_e(\omega) = \frac{\sigma^2}{2\pi}$ [10 marks]

QUESTION FIVE (20 MARKS)

Find the spectral density function of the process [20 marks]

$$e_t = X_t - 0.2X_{t-1} - 0.8X_{t-2}$$

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