

# Equilibrium Equity Premium in a Semi Martingale Market When Jump Amplitudes Follow a Binomial Distribution

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## Abstract

This paper studies equilibrium equity premium in a semi martingale market when jump amplitudes follow a binomial distribution. We take  $n$  to be the number of times. An investor is trading in this market with  $p$  being the probability that there is a shift in the price at the trading time  $t$ . We find significant variations in the equilibrium equity premium for the martingale and semi martingale markets in terms of wealth value, volatility and other parameters under study. In this market, the equilibrium equity premium remains constant regardless of volatility and wealth value.

## Keywords

Binomial Distribution, Semi Martingale, Risk Premium, Jump Diffusion

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## 1. Introduction

A semi martingale market is a partially predictable market with a decomposition

$$X_t = X_0 + M + A,$$

such that  $M = (M_t)_{0 \leq t \leq T}$  is a square-integrable martingale with  $M_0 = 0$  and  $A = (A_t)_{0 \leq t \leq T}$  being a predictable process of finite variation with  $A_0 = 0$ . This market is so attractive to investors as it is deemed fair and enables uncertainty risks to be compensated fairly. This fair compensation is usually termed as Risk Premium which of late has attracted a lot of attention from researchers. [1] and [2] had considered cases where this premium evolves according to a compensated compound poisson process in the martingale

market, but to make it more interesting, our paper considers the evolution of the model in the semi martingale market by fixing the jump amplitudes to follow a Binomial distribution.

[3] modeled a stock price as a production process in a production economy with jump diffusion and established a general equilibrium model for the equity premium. These authors proposed a pricing kernel and used it to price options. Specifically, [3] derived analytical expressions for the return distributions in the physical and the risk-neutral measures, and their model provided empirical evidence supporting the negative risk-neutral skewness and the relation between the moments of the risk-neutral and physical distributions. Their model provided more empirical evidence supporting the negative excess return of a Delta-hedged portfolio.

More recent studies [4] [5] [6] and [7] have developed an equilibrium asset and option pricing model in a production economy under jump diffusion. Their model was based on the intertemporal general equilibrium model of a production economy. Asset pricing in [8] provided analytical formulas for an equity premium and a more general pricing kernel that links the physical and risk neutral densities, which explained the two empirical phenomena of the negative variance risk premium and implied volatility smirk when a market crash is expected. [8] showed that jump size was indeed negative and that the risk aversion coefficient assumed a reasonable value when taking the jump into account. However, despite studying the systematic risk premium of the market portfolio, [8] did not model the impact of new technological developments and the dynamics of individual stock prices.

A discrete random variable (RV) is a Binomial if it arises from Bernoulli trials. There is a fixed number,  $n$ , of independent trials which by independence, means the result of any trial (for example, trial 1) does not affect the results of the following trials, and all trials are conducted under the same conditions. Under these circumstances, we define the binomial random variable  $X$  as the number of successes in  $n$  trials. The notation  $X \sim B(n, p)$  is usually used to imply that  $X$  is a random variable following a binomial distribution. The mean is  $\mu = np$  and the standard deviation is  $\sigma = \sqrt{npq}$  for some probability of failure  $q$ . The probability of exactly  $x$  successes in  $n$  trials is  $p^x q^{n-x}$ . In this market, everytime we observe a jump (shift in price), we record it as a success with probability of occurrence,  $p$ . Therefore,  $p = E(dN_t) = \lambda dt$  which studies by [9] and [10] took as the probability that  $N_t$  will jump once in the given period.

In literature, Jump Diffusion has been widely used in Option Valuation as opposed to modelling equity premium. In discrete time, the theory was proposed by [11] and later developed by [3] [4] [5] [6] [7] [12]-[17] where the Binomial option pricing model has been extensively used. More recent studies [4] [5] [6] [7] [15] [16] and [17] have developed an equilibrium asset and option pricing model in a production economy under jump diffusion. For general cases, one can see [18]-[28]. In our case, we use the binomial principles in finance to model equilibrium equity premium when the price process is a semi martingale.

Unlike [9] who fixed  $P$  to positive jumps, we take this probability whenever we observe a jump (whether positive or negative). This allows us to study the impact of jumps themselves on the equity premium.

Our contribution in this paper is comparable to [3] and also further elaboration by [9] and [10] who considered the martingale case of equilibrium equity premium. This contribution enables investors to compare the martingale and semi martingale markets in terms of the premium expected for having taken some risk in the equity market. This will make them be aware of the necessary judgements to be undertaken before one can consider investing in a partly predictable market whose price process can either go up or come down in value.

## 2. The Model

To formulate the model, we consider that the price process is arising from a Binomial distribution with one parameter  $p$ , the probability of observing a jump in a given time interval  $[0, t)$  and since the market is partially volatile, our process will evolve as a compensated compound poisson process similar to that of [3] and also further elaboration by [9] and [10].

Let's consider a Jump Diffusion process;

$$dX_t = \mu \delta dt + dB_t + (e^x - 1)dN_t - \lambda E(e^x - 1)dt$$

which is a semi martingale with discontinuities because of the presence of jumps.

We take  $\mu\delta$ , and  $\lambda$  as constants and  $x$  as a vector of jump sizes following a binomial distribution. The processes  $B_t$  and  $N_t$  are independent. This follows directly from the definition of Brownian motion as being a continuous process and the Poisson being discrete which we obviously know that continuous processes and discrete are independent.  $\lambda$  is the frequency of the Poisson process.

We set  $(e^x - 1)$  in the jump process so that  $e^x - 1 = 0$  if there is no jump as  $x$  is then a zero vector.  $E$  is the expectation which makes the process  $e^x - 1$  deterministic.  $dN_t$  models the sudden changes as a result of rare events happening and  $dB_t$  models small continuous changes generated by the noise whose volatility is a constant  $\delta$ .

The compensated compound Poisson process  $(e^x - 1)dN_t - \lambda E(e^x - 1)dt$  has the mean of zero because

$$E[(e^x - 1)dN_t - \lambda E(e^x - 1)dt] = E(e^x - 1)E(dN_t) - E(e^x - 1)E(\lambda dt) = 0 \text{ and } E(dN_t) = \lambda dt.$$

To solve

$$dX_t = \mu \delta dt + dB_t + (e^x - 1)dN_t - \lambda E(e^x - 1)dt$$

we do not need to apply Itô Lemma with Jumps because the diffusion part is a continuous semi martingale whose procedure for solution does not require the integrating factor. We solve for the price process at the terminal time  $T$  as follows:

$$dX_t = [\mu - \lambda E(e^x - 1)]dt + \delta dB_t + (e^x - 1)dN_t$$

By integration we have

$$X_T = X_t + \int_t^T [\mu - \lambda - E(e^x - 1)] \xi_t \delta + B_t + \sum_{i=1}^{N\tau} (e^{x_i} - 1), \text{ for } \tau = -T$$

as the investment period.

Suppose also that, at the risk-free rate  $\rho$ , the money market account  $X_0(t)$  is such that

$$dX_0(t) = \rho X_0(t) dt$$

whose total supply is assumed to be zero. Consider here that  $\rho$  is risk-free because it is the rate for the non risky asset (money account).

Since the value of someone's investment in this production economy at any time  $t$  is given by  $V_t = \phi X_t$ , for some portfolio  $\phi = (\omega, 1 - \omega)$  consisting of  $1 - \omega$  non risky assets and  $\omega$  risky assets, we have that by the self financing strategy,

$$dV_t = \phi dX_t$$

so that the total wealth at any time  $t$  is

$$V_t = V_0(t) + V_{t_1}(t)$$

where  $V_0(t)$  is the value of the money market account and  $V_{t_1}(t)$  is the value of the investment in the stock market at time  $t$ .

$$\begin{aligned} \text{Now } dV_t &= dV_0(t) + dV_{t_1}(t) = -(1 - \omega) dX_0(t) + \omega dX_t \\ &= -(1 - \omega) \rho X_0(t) dt + \omega [\mu - \lambda - E(e^x - 1)] \xi_t dt + \omega \delta dB_t + \omega (e^x - 1) dN_t \end{aligned}$$

Since the equity premium  $\mu - \rho = \lambda$ , we have that  $\mu - \rho = \lambda$ , hence

$$\begin{aligned} dV_t &= [(\rho X_0(t) - \omega \rho X_0(t) + \omega \phi \omega \lambda + - E(e^x - 1))] \xi_t dt \\ &\quad + \omega \delta \omega dB_t + (e^x - 1) dN_t \end{aligned}$$

The investor's optimal control problem then is to maximize his expected utility function

$$\max E_t \int_t^T y_t U r(\cdot)(\cdot) dt,$$

subject to

$$dV_t = [(\rho X_0(t) - \omega \rho X_0(t) + \omega \phi \omega \lambda +$$

$$E(e^{1^x} - -) r_t] \xi_t dt$$

$+ \omega \delta \omega dB_t + (e^{1-d^*} - 1) N_t$  The wealth ratio  $\omega$  and consumption rate  $r_t$  are control variable. The general equilibrium occur when  $\omega=1$ .

### 3. Results and Discussion

Theorem 1. In a semi martingale market with binomial jumps, an investor's equilibrium equity premium with CRRA power utility function

$U(r_t) = \frac{r_t^\beta}{\beta}$ ,  $0 < \beta < 1$ , in the production economy with jump diffusion is given by

$$\phi = X_0(t) - \rho - \beta(1 - V_t^{-1} \delta) \lambda + (1 + (e - 1) p)^n - \lambda q (1 + p p e^\beta)^n - + \lambda \lambda q (1 + p p e^{\beta-1})^n$$

where  $\phi = X_0(t) - \rho - \beta(1 - V_t^{-1} \delta)$  is the diffusive risk premium and  $\lambda_N = (1 + (e - 1) p)^n - \lambda q (1 + p p e^\beta)^n - + \lambda \lambda q (1 + p p e^{\beta-1})^n$  is the rare-event premium.

Proof. If  $X$  is a random variable with a binomial distribution, then  $Y = e^X$  is a logbinomial random variable.

In particular, if  $X \sim B(n, p)$  and  $Y = e^X$  then  $Y^k = e^{kX}$ . Also

$$E[e^{kX}] = m_X(k)$$

where  $m_X(k)$  is the moment-generating function of  $X$  evaluated at  $k$ . Hence

$$E[e^{kX}] = (1 + p p e^k)^n$$

and so

$$E[e^{kX}] = (1 + (e - 1) p)^n = m_X(1)$$

Let  $X = x$  be a vector of binomially distributed jump sizes then for the power

utility function of [10], the rare-event premium  $\phi = \lambda_N = E[(e^x - 1)^\beta (e^x)^{\beta-1}]$  which becomes

$$\phi = \lambda_N = E[e^{x + V_t e^{x + (\beta-1)}} - + 1 V_t e^{x(\beta-1)}]$$

Now, taking  $E[V_t] = q$ , we realise that:

$$E[e^{x + (\beta-1)}] = E[e^{x(1 + \beta - 1)}] = E[e^{\beta x}] = (1 + p p e^\beta)^n = m_X(\beta)$$

$$E[e^{x(\beta-1)}] = - + (1 + ppe^{\beta-1})^n = m^x(\beta-1)$$

Therefore, our rare-event premium

$$\phi \lambda_N = [E(e_x) - qE(e_{x+x(\beta-1)})] - + 1$$

$$qE(e_{x(\beta-1)})$$

now becomes  $\phi \lambda_N = [(1 + (e-1)p)^n - q(1 + pe^\beta)^n] - + 1 q(1 + pe^{\beta-1})^n$

which implies that our equity premium is now

$$\phi \rho = X_0(t) - - \rho \beta(1) V_t^{-1} \delta \lambda + (1 + -(e-1)p)^n$$

$$n \quad n - \lambda q(1 + ppe^\beta) - + \lambda$$

$$\lambda q(1 + ppe^{\beta-1})$$

Figure 1 suggests a constant equity premium regardless of how volatile the process becomes. This is a good result for investors in this market because they

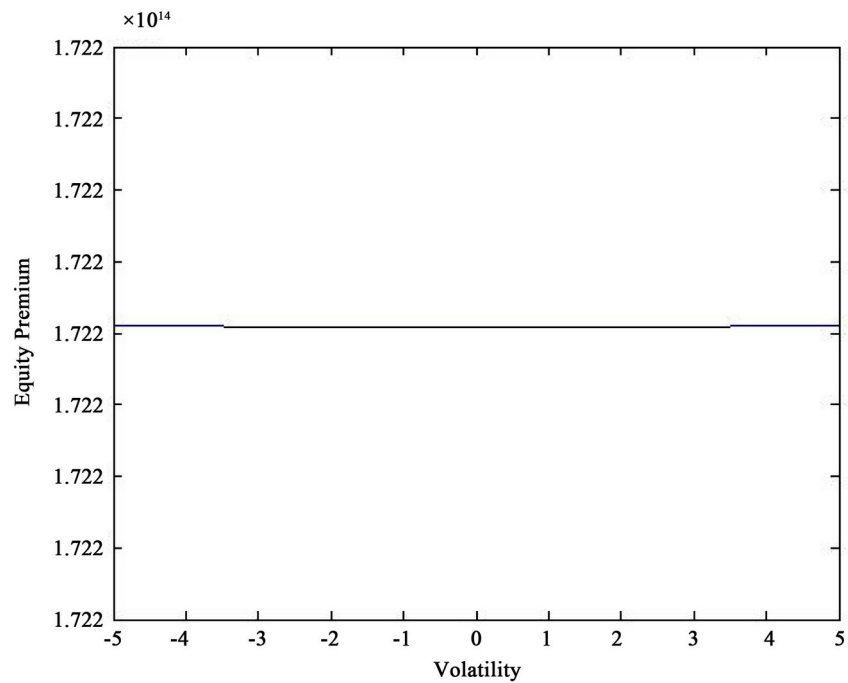


Figure 1. Power utility volatility effect under binomial distribution.

are assured of some compensation regardless of the fall or rise in the trading prices. Infact, when there is no jump expected, the premium is symmetrical about zero volatility and increases on either side (see Figure 2). This means that the diffusive risk premium always exist and is positive in this market. Therefore, the change in the price process due to time, is fairly compensated. In terms of Beta effect, it is clear from Figure 3 that the premium is zero whenever  $\beta < 2$  and decreases otherwise. This parameter affects the premium in an unattractive manner as it is inversely proportional to the equity premium on a large scale. This

is confirmed in Figure 4 as we observe an inversely proportional relationship between the equity premium and beta. This is a case when jumps are not expected. However, when jumps are expected, the compensation is fair.

Theorem 2. In the semi martingale market with binomial jumps, the investor's equilibrium equity premium with square root utility function  $U(r_t) = \kappa \sqrt{r_t}, \kappa > 0$  is given by

$$\phi \rho = \frac{\delta_2}{2V} X_0(t) - \frac{\delta_2}{2V} + \lambda \left( \frac{1 + (e^{-1})^n}{p} \right) - \lambda q \left( \frac{1 - ppe^2}{1 - ppe^2} \right) - \lambda q \left( \frac{1 - ppe^2}{1 - ppe^2} \right)$$

where  $\phi \rho = \frac{\delta_2}{2V} X_0(t) - \frac{\delta_2}{2V}$  is the diffusive risk premium and

$$\lambda_N = \left( \frac{1 + (e^{-1})^n}{p} \right) - \lambda q \left( \frac{1 - ppe^2}{1 - ppe^2} \right) - \lambda q \left( \frac{1 - ppe^2}{1 - ppe^2} \right)$$

is the rare-event premium.

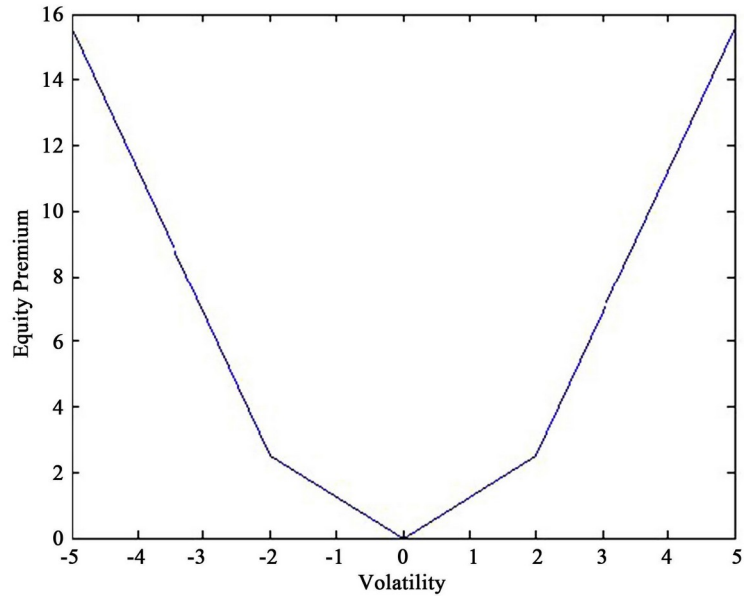


Figure 2. Power utility volatility effect under binomial distribution when no jump is expected.

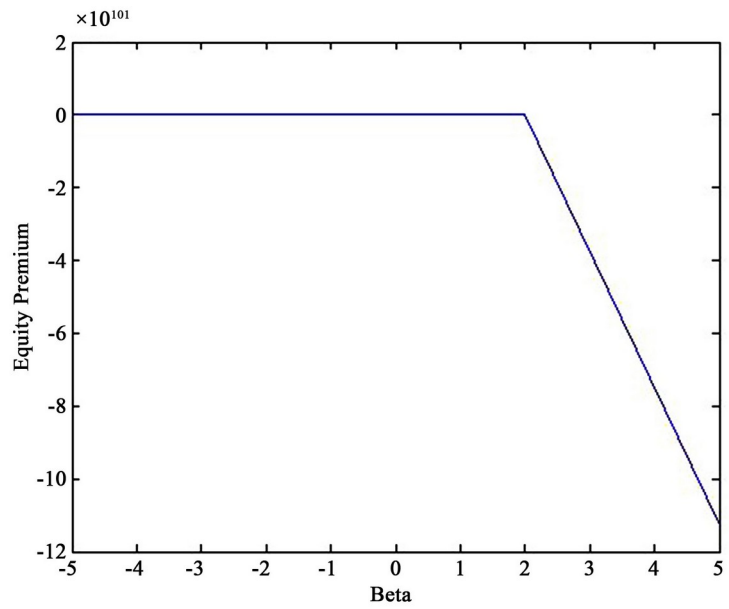


Figure 3. Power utility beta effect under binomial distribution.

Proof. For the square root utility function, the rare-event premium is given by

$$\varphi \lambda_N = E \left[ \frac{e^{x(1-\beta)} - 1}{\beta} \mid -V_t e^{x(1-\beta)} \right] \quad (\text{see [10]})$$

$$\begin{aligned} \varphi \lambda_N &= E \left[ \frac{e^{x(1-\beta)} - 1}{\beta} \mid -V_t e^{-1x} \right] \\ &= \lambda E \left[ \frac{e^{x(1-\beta)} - 1}{\beta} \mid -V_t e^{-1x} \right] \\ &= \lambda \left( E(e^{x(1-\beta)}) - E(1) + qE(e^{2x}) \right) \end{aligned}$$

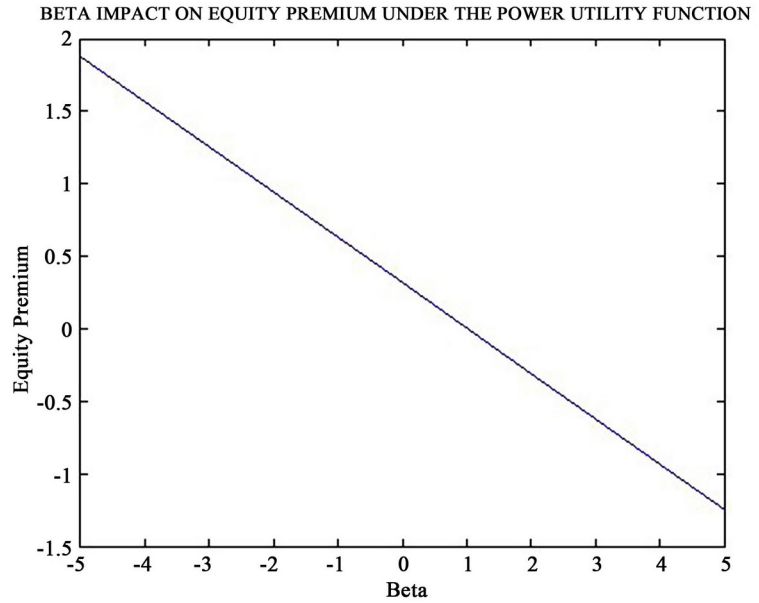


Figure 4. Power utility beta effect under binomial distribution when no jump is expected.

Since  $x \sim B n p ( , )$ , we have that

$$E [e^{x\delta}] = (1 + (e - 1) p)^n = m_x(1)$$

and

$$E [e^{x\delta} | e^{-\delta}] = (1 + p e^{-\delta})^n = m_x(e^{-\delta}) \quad (12)$$

Also

$$E [e^{-12x} | e^{-\delta}] = (1 + p e^{-12\delta})^n = m_x(e^{-12\delta})$$

Thus our rare-event premium is

$$\lambda \left( \frac{1 + (e - 1) p}{1 - q} - \frac{1 + q}{1 + p e^2} \right) e^{-\delta}$$

and therefore our equity premium is

$$\varphi = X_0(t) - \rho + \lambda \left( \frac{1 + (e - 1) p}{1 - q} - \frac{1 + q}{1 + p e^2} \right) e^{-\delta}$$

$$2V^i \left( \begin{array}{c} - \\ \end{array} \right)_n$$

$$- +\lambda \lambda q \left| 1 - +ppe^2 \right|$$

$$\left( \begin{array}{c} - \\ \end{array} \right)$$

We observe a constant premium of 1.722 in Figure 5 regardless of volatility in the process for square root utility. What is important here is that, the premium factor is positive indicating a fair compensation on the investment. However, Figure 6 is consistent with Figure 2 when no jumps are expected. These two figures confirm the positive judgement that this premium possesses in the semi

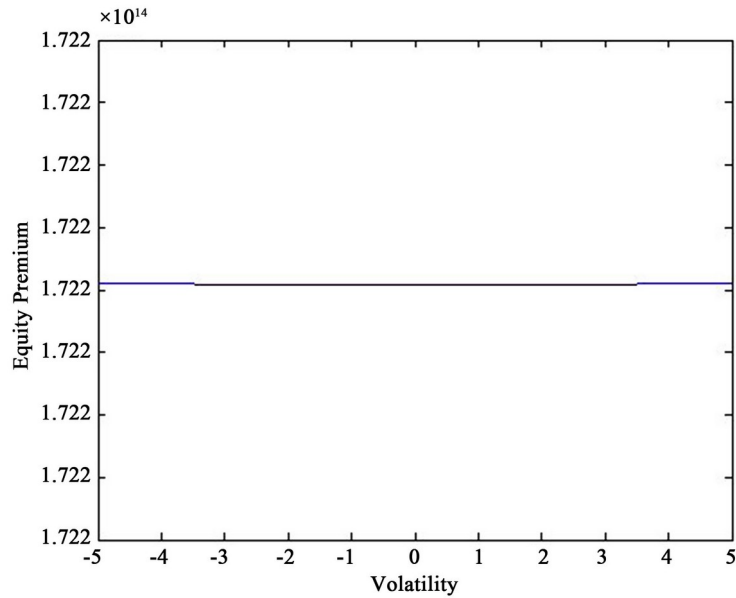


Figure 5. Square root utility volatility effect under binomial distribution.

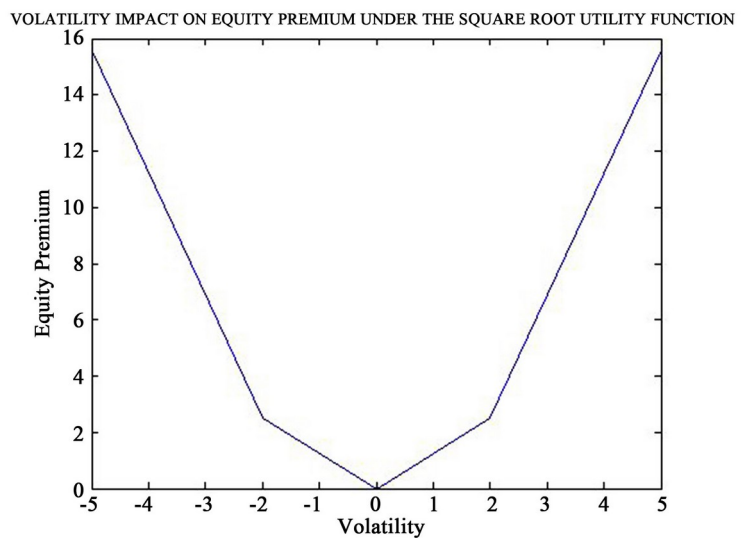


Figure 6. Square root utility volatility effect under binomial distribution when no jump is expected.

martingale market. Investors should therefore take advantage and consider investing in a market like this one.

In practice, the square root utility has many advantages in finance and economics including its ability to minimize shocks in the stock market. We were also able to see that the results for this utility function in terms of equilibrium equity premium were significantly reasonable compared to other utility functions in the martingale market.

Theorem 3. An investor's equilibrium equity premium with quadratic utility function  $U(r_t) = -r - ar a_t^2, > 0$  in the semi martingale market with normal jumps is given by

$$\begin{aligned} \varphi \rho = X_0(t) - & + \rho \frac{2a\delta^2}{1 - aV_t} + \lambda \left( (1 + (e-1)p)^n - \frac{q(1 + (e-1)p)^n}{1 - aq} \right) \\ & + \frac{2aq^2(1 + ppe^2)}{1 - aq} - \frac{1 + q - 2aq(1 + (e-1)p)}{1 - aq} \frac{q}{1 - aq} \end{aligned}$$

where  $\varphi \rho_6 = X_0(t) - + \rho \frac{2a\delta^2}{1 - aV_t}$  is the diffusive risk premium and

$$\varphi \lambda = \frac{q(1 + (e-1)p)^n - q(1 + 2(-e - aq1)p)^n + 2aq^2(1 - 21 - + paq pe^2)^n}{1 - aq}$$

is the rare-event

$$- \frac{q}{1 - aq} - \frac{2aq^2(1 + (e-1)p)^n}{1 - aq}$$

premium.

Proof. For the HARA Quadratic utility function,

$$\begin{aligned} \left( \frac{x}{1 - V_t(1 - 2aV_t e^x)} \right) \text{ so that} \\ \varphi \lambda_N = E \left[ \frac{e - 1}{1 - 2aV_t} \right] \end{aligned}$$

$$\varphi \lambda_N = E \left[ \frac{e - 1}{1 - 2aV_t} \right]$$

$$\begin{aligned}
 & \left[ \begin{array}{c} \left( \frac{V_i e^x - 2aV_i 2e^x}{1 - aV_i} - +1 \frac{V_i}{1 - aV_i} - \frac{2aV_i e^x}{1 - aV_i} \right) \\ = \lambda E \left[ e^x - \frac{V_i e^x - 2aV_i 2e^x}{1 - aV_i} - +1 \frac{V_i}{1 - aV_i} - \frac{2aV_i e^x}{1 - aV_i} \right] \\ \\ \left( \frac{V_i e^x}{1 - aV_i} + \frac{2aV_i 2e^x}{1 - aV_i} - +1 \frac{V_i}{1 - aV_i} - \frac{2aV_i e^x}{1 - aV_i} \right) \\ = \lambda E \left[ e^x - \frac{V_i e^x}{1 - aV_i} + \frac{2aV_i 2e^x}{1 - aV_i} - +1 \frac{V_i}{1 - aV_i} - \frac{2aV_i e^x}{1 - aV_i} \right] \\ \\ \left( \frac{qE(e^x)}{1 - aq} + \frac{2aqE^2(e^{2x})}{1 - aq} - +1 \frac{q}{1 - aq} - \frac{2aqE^2(e^x)}{1 - aq} \right) \\ = \lambda \left[ E(e^x) - \frac{qE(e^x)}{1 - aq} + \frac{2aqE^2(e^{2x})}{1 - aq} - +1 \frac{q}{1 - aq} - \frac{2aqE^2(e^x)}{1 - aq} \right] \end{array} \right]
 \end{aligned}$$

Now since  $x \sim B(n, p)$ , we have that

$$E[e^x] = 1 + (e - 1)p)^n = m_x(1)$$

and

$$E[e^{2x}] = 1 + (pe^2)^n = m_x(2)$$

thus our rare-event premium is

$$\begin{aligned}
 & \left[ \left( \frac{(1 + (e - 1)p)^n - q(1 + (e - 1)p)^n + 2aq^2(1 + pe^2)^n}{1 - aq} - +1 \frac{q}{1 - aq} - \frac{2aq^2(1 + (e - 1)p)^n}{1 - aq} \right) \right] \\
 & \left[ \frac{q}{1 - aq} + \frac{2aq^2(1 + (e - 1)p)^n}{1 - aq} - +1 \frac{q}{1 - aq} - \frac{2aq^2(1 + (e - 1)p)^n}{1 - aq} \right]
 \end{aligned}$$

which implies that our equity premium is

$$\begin{aligned}
 \varphi = X_0(t) - \rho & \frac{2a\delta^2}{1 - aV_i} + \lambda \left[ \frac{(1 + (e - 1)p)^n}{1 - aq} - \frac{q(1 + (e - 1)p)^n}{1 - aq} \right. \\
 & \left. + \frac{2aq^2(1 + pe^2)^n}{1 - aq} - +1 \frac{q}{1 - aq} - \frac{2aq^2(1 + (e - 1)p)^n}{1 - aq} \right]
 \end{aligned}$$

The results in Figure 7 and Figure 8 are similar to those obtained under the power utility except that, the quadratic utility is affected by the wealth process  $V_t$ . It is clear in Figure 9 that the equilibrium equity premium is constant regardless of the wealth value. However, under the quadratic utility function, the investor always receives a fair compensation for having taken some risk as long as no jump is expected. This is evident in Figure 10.

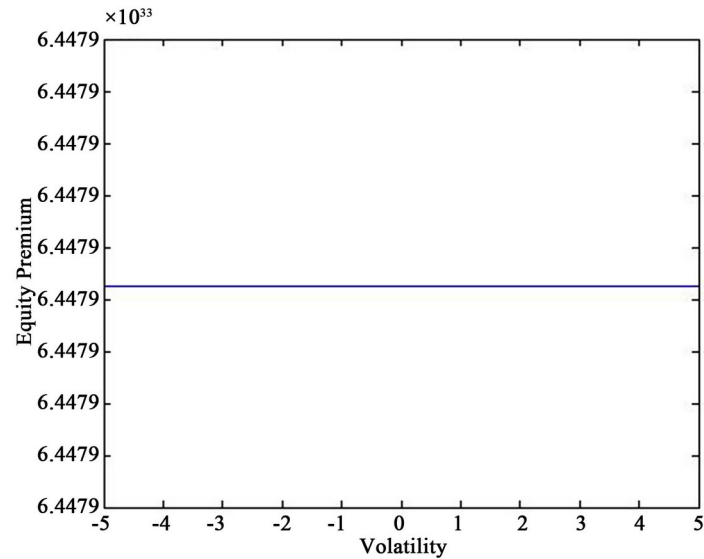


Figure 7. Quadratic utility volatility effect under binomial distribution.

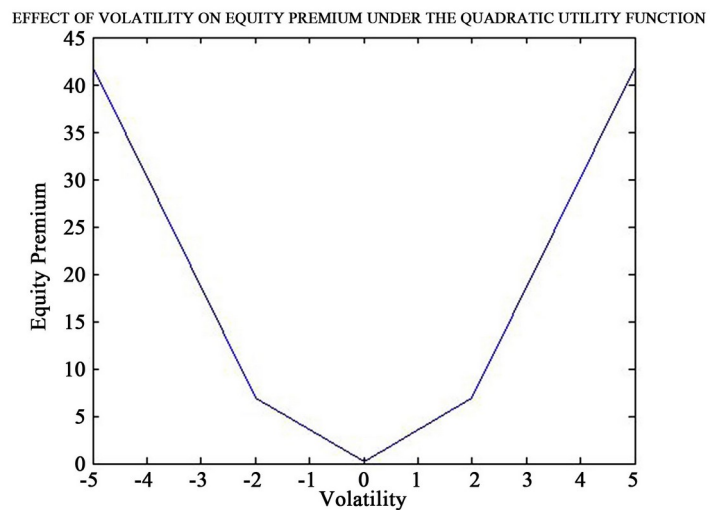


Figure 8. Quadratic utility volatility effect under binomial distribution when no jump is expected.

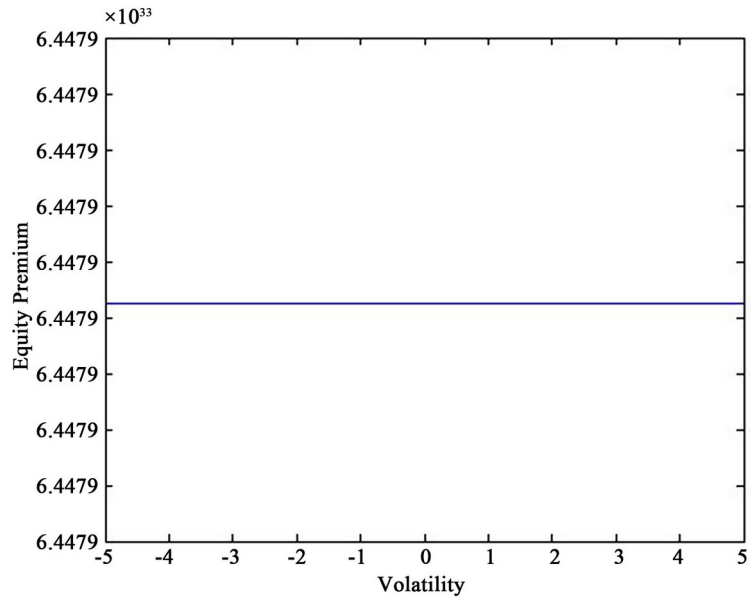


Figure 9. Quadratic utility wealth effect under binomial distribution.

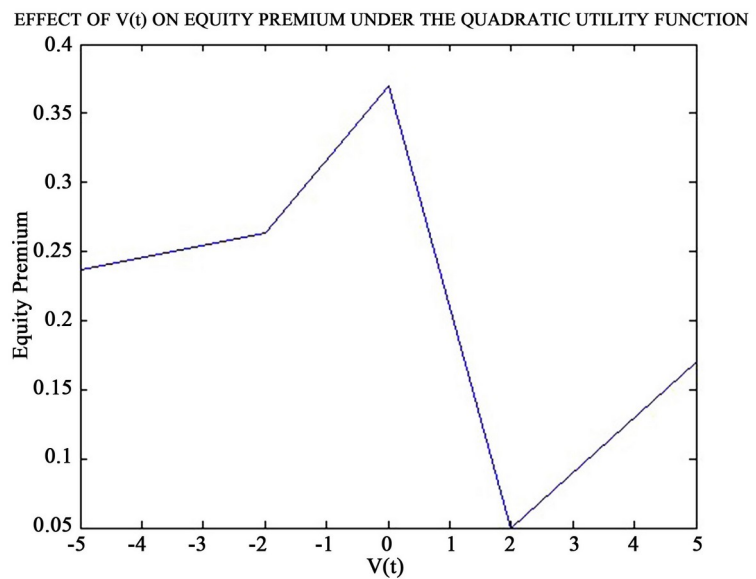


Figure 10. Quadratic utility wealth effect under binomial distribution when no jump is expected.

All in all, it is important to note that the real life processes are not martingales but prices are normalized so that the processes can then be martingales. This means that jumps must be expected in any normalized market although most scholars have generally assumed the processes without jumps. In comparison, the results in the martingale market are similar to those of the semi martingale market but the premium is not as attractive as the one realised from the semi martingale market. Infact, in this market, the equity premium is always positive regardless of the utility function the investor is following. We therefore urge investors to consider investing in this market.

## 4. Conclusions and Suggestions

The martingale and semi martingale markets differ significantly in terms of how much compensation an investor receives for having taken some risk in the investment. This is the case whenever jump amplitudes follow a binomial distribution in a semi martingale market. We observe consistent results in the equity premium of the power, square root and quadratic utility functions in terms of volatility effect, but the quadratic utility is affected also by the wealth process  $V_t$ . We therefore advise investors consuming quadratically to consider investing in the semi martingale market with jumps as long as the amplitudes follow a binomial distribution. This is to avoid external shocks and variance in the premium when jumps are not expected.

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## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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