

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN
ACTUARIAL SCIENCE

ACMT 411: COMPUTATIONAL FINANCE

STREAMS:

TIME: 2 HOURS

DAY/DATE: WEDNESDAY 18/12/2024

11.30 A.M – 1.30 P.M

INSTRUCTIONS:

Question one (30 Marks)

- A. Define a stochastic process. Define five properties of a standard Brownian Motion. (6 marks)
- B. A Safaricom share whose value is assumed to change each day according to the below probability distribution.
- +10p with probability $\frac{1}{4}$
- C + 8p with probability $\frac{1}{4}$
- 9p with probability $\frac{1}{2}$

Show that C is a martingale under the probability distribution. (5 marks)

- C. A process X_t satisfies the stochastic differential equation
 $dX_t = \sigma(x_t)dB_t + \alpha(x_t)dt$. Deduce the stochastic differential equation for the process x^3_t (4 marks)
- D. Explain the properties that a process $(x_t; t \geq 0)$ with a state space $S = \mathbb{R}$ (set of real numbers) has to achieve for it to be time homogenous. (3 marks)
- E. The Brownian motion is good at describing market indices for short term. Explain the two reasons why it cannot be used for long run prediction. (4 marks)
- F. Explain four properties of log-normal model. (8 marks)

Question two (20 marks)

- A. List and explain four assumptions of the binomial Model in asset pricing. (8 marks)
- B. Consider a binomial lattice model for a 2 month call option with an exercise price of ksh 200. Suppose that the share price either goes up by 4% or down by 3% each month, that the risk free continuously compounded rate is 0.5% per month and the current share price is also kshs 200. Estimate the value of the option. (5 marks)

- C. The continuous log-normal model is appropriate for modelling investment returns. Give seven reasons why it is not fit for modelling the returns. **(7 marks)**

Question three (20 marks)

- A. Explain the five assumptions of the black scholes model. (10 marks)
- B. A non-dividend paying share has a volatility $\delta=20\%$ p.a. Calculate the value σ and ρ for the share price movements over one month. **(5 marks)**
- C. Explain the probability that a standard Brownian Motion is confined to a region $(a-\delta, a+\delta)$ at time t in terms of a standard normal distribution. **(5 marks)**

Question 4 (20 marks)

- A. Assume that the spot rate of interest at time t , $s(t)$ can be modelled $s(t)=e^{-2\mu\omega t}$, where $w(t)$ is a Brownian Motion with drift coefficient μ and volatility coefficient σ such that $W(0)=0$.
- i) Write down an expression for $W(t)$ in terms of a standard Brownian Motion $\beta(t)$ **(4 marks)**
- ii) Show that $S(t):t>0$ is continuous time martingale. **(6 marks)**
- B. Black scholes is used for option pricing. Give and explain five reasons why it's assumptions do not hold when it comes to option **(10 marks)**

Question five (20 marks)

- A. A Company's directors have decided to provide junior managers with a performance bonus scheme. The bonus scheme entitle the managers to a cash payment of kshs 10,000 should the company's share price have increased by 20% at the end of the next six months. In addition, the managers will be entitled to kshs 5,000 free shares each, should the share price have increased by 10% at the end of the next six months.

You are given the following data

Current share price kshs 7.81

Risk free rate 5% p.a (continuously compounded)

Share price volatility 25% p.a

No dividend to be paid over the next 6 months.

- i) By considering the terms of Black scholes call option pricing formula, Calculate the value of the bonus share of the manager. **(9marks)**
- ii) Explain the main disadvantages of this bonus scheme as an incentive for managers to perform. **(5 marks)**
- iii) Some shareholders are concerned that the scheme might cause an undesirable distortion to the manager's behavior. Suggest some modification to the scheme that will ensure that the manager's aims concede with the long term objectives of the shareholders. **(6 marks)**

