CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

ACMT 311: COMPUTATIONAL METHODS AND DATA ANALYSIS III

STREAMS: BSC **TIME:2 HOURS**

DAY/DATE: TUESDAY 17/12/2024 **2.30 P.M. –4.30 P.M.**

Question one

a) Consider the uniforms 0.62 0.83 0.92 0.61 0.43 0.22 How many runs up-and-down are there? (2 marks)

b) Explain two main properties that algorithms that solve mathematical problems computationally should have (4 mark)

c) Generate 3 random variates from a Uniform(0, 5) distribution using the inverse transform method, given the random numbers 0.20,0.70. and 0.90. (6 marks)

d) The Kolmogorov-Smirnov test is used to decide if a sample comes from a population with a specific distribution.Explain three limitations of the K-S test. (6 marks)

e) Consider the two vectors below as inputed on R

```
> x <- c(TRUE,FALSE,0,6)
> y <- c(FALSE,TRUE,FALSE,TRUE)
```

What output will be dispayed from the below operators

i) `> !x` (2 marks)

ii) `> x&y` (2 marks)

iii) `> x|y` (2 marks)

f)Using Lagrange's interpolation formula find y(10) from the following table (6 marks)

x	5	6	9	11
y	12	13	14	16

Question Two

a)List any three reserved words in R programming. (3 marks)

b) Describe the Linear Congruential Generator (LCG) for generating pseudo-random numbers. Write the general formula and explain each component. (6 marks)

c) For your movies shop, the daily demand of movies with their associated probabilities is given below:

Daily demand	0	15	25	35	45	50
Probabilty	0.01	0.15	0.2	0.50	0.12	0.02

For the sequence of random numbers below 25, 39, 50, 76, 12, 02, 61, 89, 94, 42 what will the average daily demand will be? (5 marks)

d) Briefly discuss any three Numerical Analysis Software Packages highlighting their major strengths. (6 marks)

Question Three

a) Define the below terms as used in Data Analysis

i) Simulation. (2 marks)

ii) Pseudo random Number. (2 marks)

iii) Algorithm. (2 marks)

b) With the appropriate formulae explain the algorithm for generating Normal random variates using polar method. (6 marks)

c) Explain four physical devices used for generating uniform random variables. (8 marks)

Question Four

a) Describe two statistical tests that can be used to evaluate the quality of a random or pseudo-random number generator. (4 marks)

b) Given the elements 7, 21, 14, 2, 1, 19, 100, 89, 35, 12,

i. Write an R function that creates a vector x with the above elements . (2 marks)

ii. Write a function that accesses the first element of a numeric vector and print the output.(2 marks)

iii. Write a function that accesses all but 1st element and print the output. (2 marks)

iv. Write a function that finds the range of the vector and print the output. (2 marks)

v. Write a function that sorts your vector in ascending order and print the output.?

(2 marks)

c) Generate 4 random variates from a Normal(0, 1) distribution using the inverse transform method with the Box-Muller method, given the random numbers u_1=0.25and u_2=0.75 for the first pair, and u_3=0.4 and u_4=0.6 for the second pair. (6marks)

Question Five

a) What is the distribution of $X = -(1/3) \ln (1 - U)$? (2 marks)

b) A random variable x follows an exponential distribution with a rate parameter λ=2. Derive the formula for generating random samples using the inverse transform technique and use it to generate a sample with U=0.25. (5 marks)

c) You are conducting a simulation study to estimate the value of a parameter with an acceptable error margin of 0.1. If the variability in the data is estimated to be 2, how many simulations are required to achieve this error margin with 95% confidence? (5 marks)

d) Using the acceptance-rejection method,generate random variates from a distribution with probability density function defined as follows:

$$f(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & Otherwise \end{cases}$$

Where the proposal distribution g(x) = 1 for $0 \le x \le 1$ (a uniform distribution). Assume that the constant c is chosen as 2, so that $c.g(x) \ge f(x)$ for all x.

Use the random numbers 0.4, 0.2, and 0.8 (for generating candidate values) and 0.3, 0.6, and 0.7 (for acceptance-rejection decisions), determine whether each candidate is accepted or rejected.

(8 marks)
