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ABSTRACT

In this paper, we investigate the primitivity action properties of the cartesian product of an alternating group $A_n (n\geq5)$ acting on a cartesian product of ordered sets of triples using the definition primitivity and blocks. When $n\geq5$, the cartesian product of the alternating group, $A_n \times A_n \times A_n$, acts imprimitively on a cartesian product of ordered sets of triples, $P^{[3]} \times S^{[3]} \times V^{[3]}$.

Keywords: transitivity action, primitivity action, ordered sets of triples, cartesian product and alternating group.

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In this paper, we investigate the primitivity action properties of the cartesian product of an alternating group $A_n (n \geq 5)$ acting on a cartesian product of ordered sets of triples using the definition primitivity and blocks. When $n \geq 5$, the cartesian product of the alternating group, $A_n \times A_n \times A_n$, acts imprimitively on a cartesian product of ordered sets of triples, $P^{[3]} \times S^{[3]} \times V^{[3]}$.

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I. PRELIMINARIES

1.1 Notation and Terminology

In this paper, we shall represent the following notations as: $\sum$- sum over i; A_n -an alternating group of degree n and order $\frac{n!}{2}$; $|G|$ – the order of a group G; $|G: H|$ -Index of H in G;

$P^{[3]}$ – the set of an ordered triple from set $P = \{1, 2, 3,..., n\}$; $S^{[3]}$ – the set of an ordered triple from set $S = \{n + 1, n + 2,..., 2n\}$; $V^{[3]}$ – the set of an ordered triple from set

$V = \{2n + 1, 2n + 2,..., 3n\}$; $[a, b, c]$ -Ordered triple; $A_n \times A_n \times A_n$ -Cartesian product of alternating group A_n; $P^{[3]} \times S^{[3]} \times V^{[3]}$ -Cartesian product of ordered sets of triples $P^{[3]}$, $S^{[3]}$ and $V^{[3]}$.

Definition 1.1.1: Group action (Kinyanjui et al., 2013): Let P be a non-empty set. A group G is said to act on the left of P if for each $g \in G$ and each $p \in P$ there corresponds a unique element $gp \in G$ such that:

(i) $(g_1 g_2) = g_1(g_2 p)$, g_1, $g_2 \in G$ and $p \in P$.

(ii) For any $p \in P$, $ep = P$, where e is the identity in G.

The action of G from the right on P can be defined in the same manner.

Definition 1.1.2: Orbit (Njagi, 2016): Let G act on a set P. Then P is partitioned into disjoint equivalent classes called orbits or transitivity classes of the action. For every $p \in P$ the orbit containing p is called the orbit of p and is denoted by $Orb_G(p)$.

Definition 1.1.3 Fixed point (Kinyanjui et al., 2013): Let G act on a set P. The set of elements of P fixed by $g \in G$ is called the fixed-point set of G and is denoted by $Fix(g)$. Thus $Fix(g) = \{gp = p\}$.

Definition 1.1.4: Transitive group (Cameron, 1970): If the action of a group G on set P has only one orbit, then we say that G acts transitively on P. In other words, G acts transitively on P if for every pair of points $p, s \in P$, there exists $g \in G$ such that $gp = s$.

Theorem 1.1.4: (Orbit – Stabilizer Theorem, Rose, 1978, p.72): Let G act on a set P. Then $\left|Orb_G(p)\right| = \left|G: Stab_G(p)\right|$.

Definition 1.1.5: Blocks and primitivity (Nyaga et al., 2011): Assume that the action of G on X is transitive. For every subset A of X such that each $g \in G$, let $gA = \{ga: a \in A\} \subseteq X$. A subset A of X is referred to as a block for the action if, for every $g \in G$, either $gA = A$ or $gA \cap A = \emptyset$. $\emptyset, X$ and all singleton subsets of X are definitely blocks known as trivial blocks. If these are the only blocks, then G acts primitively on X otherwise, G acts imprimitively.

Definition 1.1.6: Direct product action (Cameron et al, 2008): Let (G_1, P_1) and (G_2, P_2) be permutation groups. The direct product $G_1 \times G_2$ acts on the disjoint union $P_1 \cup P_2$ by the rule $p(g_1, g_2) = \{pg_1, \text{ if } p \in P_1, pg_2, \text{if } p \in P_2$ and on the Cartesian product $P_1 \times P_2$ by the rule $(p_1, p_2)(g_1, g_2) = (p_1 g_1, p_2 g_2)$.

Theorem 1.1.7: (Armstrong, 2013): The $G_1 \times G_2 \times G_3$-orbit containing $(p, s, v) \in P \times S \times V$ is given by $Orb_{G_1}(p) \times Orb_{G_2}(s) \times Orb_{G_3}(v)$ and the stabilizer of (p, s, v) is given by $Stab_{G_1}(p) \times Stab_{G_2}(s) \times Stab_{G_3}(v)$.

1.2 Introduction

Higman (1964) introduced the rank of a group on finite permutation groups of rank 3. Cameron (1972) worked on the suborbits of multiply transitive permutations and later in 1973 studied the suborbits of primitive groups.

Hamma and Aliyu (2010), on transitivity and primitivity of dihedral groups proved that the dihedral group of degree $2^n (n \geq 2)$ is transitive and primitive. Ndarinyo *et al.*, (2015) showed that the alternating group $A_n = 5, 6, 7$ acts transitively on unordered and ordered triples from the set $P = 1, 2, \ldots, n$ when $n \leq 7$ through determination of the number of orbits.

Muriuki *et al.*, (2017) showed that for the action of direct product of three symmetric groups on Cartesian product of three sets, the action is both transitive and imprimitive for all $n \geq 2$ and the associated rank is 2^3.

Mutua *et al.*, (2018) showed that the direct product of $S_n \times A_n$, of the symmetric group S_n by the alternating group A_n on the cartesian product $X \times Y$ has its action both transitive and imprimitive when $n \geq 3$. Nyaga (2018) proved that the direct product action of the alternating group on the Cartesian product of three sets is transitive.

Maraka *et al.*, (2021) showed that the action of the cartesian product of the alternating group, $A_n \times A_n \times A_n$, on the cartesian product of $P^{[3]} \times S^{[3]} \times V^{[3]}$, the cartesian product of ordered sets of triples is transitive when $n \geq 5$.

Based on these results we investigate some properties of $A_n \times A_n \times A_n$, the cartesian product action of the alternating group acting on $P^{[3]} \times S^{[3]} \times V^{[3]}$, the cartesian product of ordered sets of triples.

The cartesian product of alternating group $A_n \times A_n \times A_n$, acts on $P^{[3]} \times S^{[3]} \times V^{[3]}$, by the rule;

$g_1\{([1,2,]3,[1,2,]4,..[k,,k-1,k-3],[k,k-1,k-2])\} \times g_2\{([k+1,k+2,k+3],[k+1,k+2,k+4],..[2k,,2k-1,2k-3],[2k,2k-1,2k-2])\} \times g_3\{([2k+1,2k+2,2k+3],[2k+1,2k+2,2k+4],...,[3k,3k-1,3k-3],[3k,3k-1,3k-3])\} = \{g_1([1,2,]3,[1,2,]4,..[k,,k-1,k-3],[k,k-1,k-2]), g_2([k+1,k+2,k+3],[k+1,k+2,k+4],..[2k,,2k-1,2k-3],[2k,2k-1,2k-2]), g_3([2k+1,2k+2,2k+3],[2k+1,2k+2,2k+4],...,[3k,3k-1,3k-3],[3k,3k-1,3k-3])\}$;

$\forall\, g_1, g_2, g_3 \in A_n, \{([1,2,]3,[1,2,]4,...,[k,k-1,k-3],[k,k-1,k-2])\} \in P^{[3]}$, set of ordered triples from{t1h,e2,s3e,t..};$P\{,\notin[$ +1, $k+2, k+3],[k+1,k+2,k+4],..[2k,,2k-1,2k-3],[2k,2k-1,2k-2])\} \in S^{[3]}$, set of ordered triples from{tkh+e1,sekt+2,S.=..},;a2nkd$\{([2k+1,2k+$

II. MAIN RESULTS

Theorem 2.1: (Maraka et. al., 2021): The action of the cartesian product of the alternating group A_n, $A_n \times A_n \times A_n$, acting on the cartesian product of ordered sets of triples, $P^{[3]} \times S^{[3]} \times V^{[3]}$, is transitive if and only if $n \geq 5$.

Proof: Let $G = G_p \times G_s \times G_v = A_n \times A_n \times A_n$ act on $P^{[3]} \times S^{[3]} \times V^{[3]}$. It suffices to verify that $|P^{[3]} \times S^{[3]} \times V^{[3]}|$ is equal to $|Orb_G([1,2,3],[k+1,k+2,k+3],[2k+1,2k+2,2k+3]).|$

$$\text{Let } |R| = |Stab_G([1,2,3],[k+1,k+2,k+3],[2k+1,2k+2,)2|.k+3]$$

So, $(g_p, g_s, g_v) \in G = A_n \times A_n \times A_n$ fixes $([1,2,3],[k+1,k+2,k+3],[2k+1,2k+2,2k+3]) \in P^{[3]} \times S^{[3]} \times V^{[3]}$ if and only if 1, 2 and 3 comes from 1-cycle of g_p; $k+1$, $k+2$ and $k+3$ comes from 1-cycle of g_s and $2k+1$, $2k+2$ and $2k+3$ comes from 1-cycle of g_v.

$$\text{Therefore, } |R| = |Stab_G([1,2,3],[n+1,n+2,n+3],[2n+1,2n+2,2n+3])|$$

$$= |Stab_{G_p}([1,2,3]) \times Stab_{G_s}([n+1,n+2,n+3]) \times Stab_{G_v}([2n+1,2n+2,2n+3])|$$

$$|R| = \frac{(k-3)! \times (k-3)! \times (k-3)!}{2 \times 2 \times 2} = \left(\frac{(k-3)!}{2}\right)^3$$

Applying the Orbit-Stabilizer Theorem we get;

$$|Orb_G([1, 2, 3], [k+1, k+2, k+3], [2k+1, 2k+2,)2|k+3]$$

$$= |G: Stab_G([1, 2, 3], [k+1, k+2, k+3], [2k+1, 2k+2,)2|k+3]$$

Therefore;

$$|G| = \frac{k! \times k! \times k!}{2 \times 2 \times 2} = \left(\frac{k!}{2}\right)^3$$

$$\frac{|G|}{|R|} = \left(\frac{k!}{(k-3)!}\right)^3 = |P^{[3]} \times S^{[3]} \times V^{[3]}|$$

Hence, $A_n \times A_n \times A_n$ acts transitively on $P^{[3]} \times S^{[3]} \times V^{[3]}$ if $n \geq 5$.

Lemma 2.2: The action of the cartesian product of the alternating group A_5, $A_5 \times A_5 \times A_5$, acting on the cartesian product of ordered sets of triples, $P^{[3]} \times S^{[3]} \times V^{[3]}$, is imprimitive.

Proof: This action is transitive by Theorem 2.1.

Now, for $n = 5$, the set $P = \{1, 2, 3, 4, 5\}$, so, $gap > Arrangements([1, 2, 3, 4, 5], 3);$

$P^{[3]} = \{[1, 2, 3], [1, 2, 4], \ldots, [5, 4, 1],; [5, 4, 2], [5, 4, 3]\}$

$S = \{6, 7, 8, 9,\},1g0ap>$ Arrangements([6,7,8,9,10],3);

$S^{[3]} = \{[6, 7, 8], [6, 7, 9], [6, 7, 10], \ldots, [10, 9, a6n d], [10, 9, 7], [10, 9,$

$V = \{11, 12, 13, 14,\},1s5o$ gap> Arrangements([11,12,13,14,15],3);

$V^{[3]} = \{[11, 12, 13], [11, 12, 14], \ldots, [15, 14, 11], [15,. 14, 12], [15, 14, 13]$

We have;

$K =$

$\{([1, 2,]3, [1, 2,]4 [1, 2,]5 \ldots,[5, 4,]1 [5, 4,]2 [5, 4,]3) \times$

$([6, 7,]8[6, 7,]9[6, 7, 1]0 \ldots,[10, 9,]6[10, 9,]7[10, 9,]8) \times$

$([11, 12, 1]3 [11, 12, 1]4 [11, 12, 1]5 \ldots,[15, 14, 1]1 [15, 14, 1],2[15, 14, 1]3)\}$

Let K' be the non-trivial subset of K, $K' = P^{[3]^F} \times S^{[3]^F} \times V^{[3]^F}$ such that $|K'|$ divides $|K|$.

So, we have; $\dfrac{\frac{(5-3)!}{} \times \frac{(5-3)!}{} \times \frac{(5-3)!}{}}{\frac{5!}{(5-3)!}} = \dfrac{|K|}{|K^F|}$

Now,

$K' =$

{ ([1, 2, 3] , [6, 7, 8] , [ )11, ..1, 2([, 1,42], 3] , [6, 7, 8] , [ )1, 5([, 1,42 ,133] , [6, 7, 9] , [1)1}, 12, 13]

therefore, $|K'| = \frac{5!}{(5-3)!}$.

For each element of K' there exist $(g_p, g_s, g_v) cG$ with $\frac{5!}{(5-3)!}$ cycles permutation,

{([1, 2, 3] , . . . , [([5, 64, , 73, 8])], , ... ,)[, ([1 01 19,, 182,]13] , . . . , [ )1 5, s 1u c4h, 13]

that for every $(g_p, g_s, g_v) \epsilon G$; [1, 2, 3] is fixed in g_p , [6, 7, 8] is fixed in g_s and that [11, 12, 13] belongs to a single cycle of g_v , then, (g_p, g_s, g_v) either fixes an element of $K' = P^{[3]'} \times S^{[3]'} \times V^{[3]}$ or takes one element of K' to another so that; $(g_p, g_s, g_v)K' = K'$. Any other $(g_p, g_s, g_v) \epsilon G$ moves an element of K' to an element not in K' so that; $(g_p, g_s, g_v) K' \cap K' = \emptyset$. Thus, K' is a non-trivial block for the action and it follows from definition 1.1.5 that the action is imprimitive.

Lemma 2.3: The action of the cartesian product of the alternating group A_6, $A_6 \times A_6 \times A_6$, acting on the cartesian product of ordered sets of triples, $P^{[3]} \times S^{[3]} \times V^{[3]}$, is imprimitive.

Proof: This action is transitive by Theorem 2.1.

Now, for $n = 6$, the set $P = \{1, 2, 3, 4, 5, 6\}$, so, gap> Arrangements([1,2,3,4,5,6],3);

$P^{[3]}$ ={ [1, 2, 3] , [1, 2, 4] , [1, 2, 5] , . . . , [6, 5, ;2] , [6, 5, 3] , [6, 5, 4]}

$S = \{7, 8, 9, 10, 11\}$,, s1o2gap> Arrangements([7,8,9,10,11,12],3);

$S^{[3]}$ ={ [7, 8, 9] , [7, 8, 10] , [7, 8, 11] , . . . , [12, 11, 8] , a[nd12, 11, 9] , [12, 1

$V = \{13, 14, 15, 16, 17\}$,, s1o8gap> Arrangements([13,14,15,16,17,18],3);

$V^{[3]} =$

{ [13, 14, 15] , [13, 14, 16] , [13, 14, 17] , . . . , [18, 17, 14] , .[18, 17, 15] , [

We have;

$K =$

{([1, 2, 3] , [1, 2, 4] , [1, 2, 5] , . . . , [6, 5, 2] , [6, 5, 3] , [6, 5, 4]) ×

([7, 8, 9] , [7, 8, 10] , [7, 8, 11] , . . . , [12, 11, 8]), ×[12, 11, 9] , [12, 11, 10

([13, 14, 15] , [13, 14, 16] , [13, 14, 17] , . . . , [18, 17, 14] ,)[} 18, 17, 15] , [

Let K' be the non-trivial subset of K, $K' = P^{[3]^F} \times S^{[3]^F} \times V^{[3]^F}$ such that $|K'|$ divides $|K|$.

We have; $\frac{\frac{6!}{(6-3)!} \times \frac{6!}{(6-3)!} \times \frac{6!}{(6-3)!}}{\frac{6!}{(6-3)!}} = \frac{|K|}{|K^F|}$

Now,

$K' =$

{([1, 2, 3] , [7, 8, 9] , [ )13, ...1,(4[, 11 , 62], 3] , [7, 8, 9] , [ )1,8([, 11,72,, 136], [7, 8, 10] , [1)3}, 14, 15]

therefore, $|K'| = \frac{6!}{(6-3)!}$.

For each element of K' there exist $(g_p, g_s, g_v)\epsilon G$ with $\frac{6!}{(6-3)!}$ cycles permutation, {([1, 2, 3],..., [[6, 5, 4]), ([7, 8, 9],, ..., [12, 11, 10]), ([13, 14, 15],..., [18, 17, 16])} such that for every $(g_p, g_s, g_v)\epsilon G$; [1, 2, 3] is fixed in g_p, [7, 8, 9] is fixed in g_s and that [13, 14, 15] belongs to a single cycle of g_v, then, (g_p, g_s, g_v) either fixes an element of $K' = P^{[3]'} \times S^{[3]'} \times V^{[3]'}$ or takes one element of K' to another so that; $(g_p, g_s, g_v)K' = K'$. Any other $(g_p, g_s, g_v)\epsilon G$ moves an element of K' to an element not in K' so that; $(g_p, g_s, g_v)K' \cap K' = \emptyset$. Thus, K' is a non-trivial block for the action and it follows from definition 1.1.5 that the action is imprimitive.

Lemma 2.4: The action of the cartesian product of the alternating group A_7, $A_7 \times A_7 \times A_7$, acting on the cartesian product of ordered sets of triples, $P^{[3]} \times S^{[3]} \times V^{[3]}$, is imprimitive.

Proof: This action is transitive by Theorem 2.1.

Now, for $n = 7$, the set $P = \{1, 2, 3, 4, 5, 6, 7\}$, so, gap> Arrangements ([1,2,3,4,5,6,7],3);

$P^{[3]}$ ={ [1, 2, 3] , [1, 2, 4] , [1, 2, 5] , . . . , [7, 6, ;3] , [7, 6, 4] , [7, 6, 5] }

S = {8, 9, 10, 11, 12, 1}3, ,so1g4ap> Arrangements([8,9,10,11,12,13,14],3);

$S^{[3]}$ ={ [8, 9, 10] , [8, 9, 11] , [8, 9, 12] , . . . , [14, 13, 10] , [an1d4 , 13, 11] , [1

V = {15, 16, 17, 18, 19, 2}0 ,so2g1ap> Arrangements([15,16,17,18,19,20,21],3); $V^{[3]}$ =

{[15, 16, 17] , [15, 16, 18] , [15, 16, 19] , . . . , [21, 20, 17] , [. 21, 20, 18] , [

We have;

$K =$

{([1, 2, 3] , [1, 2, 4] , [1, 2, 5] , . . . , [7, 6, 3] , [7, 6, 4] , [7, 6, 5]) ×

([8, 9, 10] , [8, 9, 11] , [8, 9, 12] , . . . , [14, 13, 10] , [) ×14, 13, 11] , [14, 13,

([15, 16, 17] , [15, 16, 18] , [15, 16, 19] , . . . , [21, 20, 17] ,)[} 21, 20, 18] , [2

Let K' be the non-trivial subset of K, $K' = P^{[3]^F} \times S^{[3]^F} \times V^{[3]^F}$ such that $|K'|$ divides $|K|$

so, we have; $\frac{\frac{(7-3)!}{7!} \times \frac{(7-3)!}{\frac{77!!}{(7-3)!}} \times \frac{(7s-3)!}{7!}}{} = \frac{|K|}{|K^F|}$.

$K' =$

{([1, 2, 3] , [8, 9, 10] , [1)5,, ..1 ,6([, 1,82], 3] , [8, 9, 10] , [2) ,1(,[12,02 ,139]], [8, 9, 11] , [1)5}, 16, 17]

therefore, $|K'| = \frac{7!}{(7-3)!}$.

For each element of $K^{'}$ there exist $\left(g_p, g_s, g_v\right) \epsilon G$ with $\frac{7!}{(7-3)!}$ cycles permutation, $\{([1,2,3],\ldots,[7,6,5]),([8,9,10],,\ldots,[14,13,12]),([15,16,17],\ldots,[21,20,19])\}$ such that for every $\left(g_p, g_s, g_v\right) \epsilon G$; $[1,2,3]$ is fixed in g_p, $[8,9,10]$ is fixed in g_s and that $[15,16,17]$ belongs to a single cycle of g_v, then, $\left(g_p, g_s, g_v\right)$ either fixes an element of $K^{'} = P^{[3]^{'}} \times S^{[3]^{'}} \times V^{[3]^{'}}$ or takes one element of $K^{'}$ to another so that; $\left(g_p, g_s, g_v\right) K^{'} = K^{'}$. Any other $\left(g_p, g_s, g_v\right) \epsilon G$ moves an element of $K^{'}$ to an element not in $K^{'}$ so that; $\left(g_p, g_s, g_v\right) K^{'} \cap K^{'} = \emptyset$. Thus, $K^{'}$ is a non-trivial block for the action and it follows from definition 1.1.5 that the action is imprimitive.

Theorem 2.5: The action of the cartesian product of the alternating group A_n, $A_n \times A_n \times A_n$, acting on the cartesian product of ordered sets of triples, $P^{[3]} \times S^{[3]} \times V^{[3]}$, is imprimitive for $n \geq 5$.

Proof: The action of $A_n \times A_n \times A_n$ on $P^{[3]} \times S^{[3]} \times V^{[3]}$ is transitive by Theorem 2.1 for $n \geq 5$.

Consider, $\forall\ g_1, g_2, g_3 \in A_n$, $\{([1,2,3],[1,2,4],\ldots,[n,n-1,n-3],[n,n-1,n-2])\} \in P^{[3]}$, set of ordered triples from the set $P = \{1,2,3,\ldots,n\}$; $\{([n+1,n+2,n+3],[n+1,n+2,n+4],\ldots,[2n,2n-1,2n-3],[2n,2n-1,2n-2])\} \in S^{[3]}$,

set of ordered triples from the set

$S = \{n+1, n+2, \ldots, 2n\}$; and $\{([2n+1,2n+2,2n+3],[2n+1,2n+2,2n+4],\ldots,[3n,3n-1,3n-3],[3n,3n-1,3n-2])\} \in V^{[3]}$,

set of ordered triples from the set $V = \{2n+1, 2n+2, \ldots, 3n\}$.

We have;

$K = \{([1,2,3],[1,2,4],\ldots,[k,k-1,k-3],[k,k-1,k-2]) \times ([k+1,k+2,k+3],[k+1,k+2,k+4],\ldots,[2k,2k-1,2k-3],[2k,2k-1,2k-2]) \times ([2k+1,2k+2,2k+3],[2k+1,2k+2,2k+4],\ldots,[3k,3k-1,3k-3],[3k,3k-1,3k-3])\}$.

Let $K^{'}$ be the non-trivial subset of K; $K^{'} = P^{[3]^{'}} \times S^{[3]^{'}} \times V^{[3]^{'}}$ such that $\left|K^{'}\right|$ divides $|K|$.

$$\text{Therefore;} \quad \frac{\frac{n!}{(n-3)!} \times \frac{n!}{(n-3)!} \times \frac{n!}{(n-3)!}}{\frac{n!}{(n-3)!}} = \frac{|K|}{|K^{F}|}$$

$$\text{Now, } K' = \left\{ \begin{matrix} ([1,2,3],[k+1,k+2,k+3],[2k+1,2k+2,2k+4]),\ldots, \\ ([1,2,3],[k+1,k+2,k+3],[3k,3k-1,3k-3]), \\ ([1,2,3],[k+1,k+2,k+4],[2k+1,2k+2,2k+3]) \end{matrix} \right\}.$$

So, $|K'| = \frac{n!}{(n-3)!}$

For every element of K' there exist $(g_p, g_s, g_v) \epsilon G$ with $\frac{n!}{(n-3)!}$ cycles permutation;

$\{([1,2,]3,[1,2,]4 \ldots [k,, k-1, k-3], [k, k-1, k-2]), ([k+1, k+2, k+3], [k+1, k+2, k+4], \ldots [2k,, 2k-1, 2k-3], [2k, 2k-1, 2k-2]), ([2k+1, 2k+2, 2k+3], [2k+1, 2k+2, 2k+4], \ldots, [3k, 3k-1, 3k-3], [3k, 3k-1, 3k-3])\}$ such that for every $(g_p, g_s, g_v) \subset G$; $[1, 2,]3$ is fixed in g_p, $[k+1, k+2, k+3]$ is fixed in g_s and that $[2k+1, 2k+2, 2k+3]$ belongs to a single cycle of g_v, then, (g_p, g_s, g_v) either fixes an element of $K' = P^{[3]^F} s \times S^{[3]^F} \times V^{[3]^F}$ or takes one element of K' to another so that; $(g_p, g_s, g_v) P^{[3]^F} \times S^{[3]^F} \times V^{[3]^F} = P^{[3]^F} \times S^{[3]^F} \times V^{[3]^F}$. Any other $(g_p, g_s, g_v) \subset G$ moves an element of K' to an element not in K' so that; $(g_p, g_s, g_v) P^{[3]^F} \times S^{[3]^F} \times V^{[3]^F} \cap P^{[3]^F} \times S^{[3]} \times V^{[3]} = \emptyset$. This argument shows that K is a non-trivial block for the action and the conclusion follows from definition 1.1.5 hence the action is imprimitive for $n \geq 5$.

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