

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN
FOOD SCIENCE AND TECHNOLOGY

CHEM 304: ANALYTICAL CHEMISTRY I

STREAMS: BSC

TIME: 2 HOURS

DAY/DATE: MONDAY 23/12/2024

8. 30 A.M – 10.30 A.M.

INSTRUCTIONS

- Answer question One (Compulsory) and any other Two questions.

Useful data

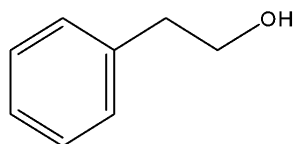
$$c = 3.0 \times 10^8 \text{ m/s}$$

$$h = 6.626 \times 10^{-34} \text{ J.S}$$

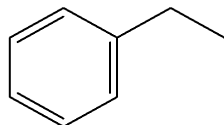
QUESTION ONE (30 MARKS)

- a) Differentiate between the following:
- (i) Adsorption and partition chromatography (2 marks)
 - (ii) Bathochromic and hypsochromic shift (2 marks)
- b) Explain the principle of size exclusion chromatography. (3 marks)
- c) A 0.1 M solution of acid was used to titrate 10 ml of 0.1 M solution of alkali and the following volumes of acid were recorded:
9.88, 10.18, 10.23, 10.39, 10.21 ml
Calculate the 95% confidence limits of the mean and use them to decide whether there is any evidence of systematic error. (5 marks)
- d) State the advantages of inductively coupled plasma compared with flame in atomic spectroscopy. (2 marks)

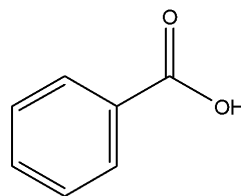
- e) Based solely on molecular polarity, predict the most likely order of elution for the following compounds in a reversed phase chromatographic experiment. Justify your choice of the order. (2 marks)



2-phenylethanol

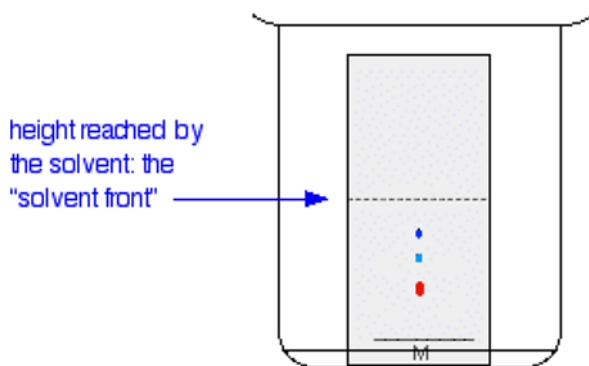


ethylbenzene



benzoic acid

- f) Briefly explain how temperature programming improves separation in gas chromatography (GC). (3 marks)
- g) The following diagram shows the results of a simple thin layer chromatography (TLC) experiment.



- (i) Describe briefly the procedure that would have been followed in order to obtain the above TLC chromatogram, assuming that a suitable thin layer chromatography plate was provided. (2 marks)
- (ii) Describe briefly how the retardation factor (R_f) value for each of the spots on the above chromatogram would be determined. (1 mark)
- (iii) TLC technique can be used to identify particular amino acids in a mixture of amino acids. However, the chromatograms obtained are colourless. State a staining reagent that can be used to spray the plate to make the spots visible. (1 mark)
- h) In a hydrogen-oxygen flame, the atomic absorption signal for iron decreases in the presence of large concentrations of sulfate ions.
- (i) Suggest an explanation for this observation. (1 mark)
- (ii) Discuss two methods that can be used to overcome the potential interference of sulfate in a quantitative determination of iron. (4 marks)
- i) A sample of pure vitamin A has $\lambda_{\text{max}} = 325 \text{ nm}$. Its molar absorptivity is $50,100 \text{ M}^{-1}\text{cm}^{-1}$ and the path length is 1 cm. Using a spectrophotometer, the obtained value of absorbance at 325 nm is 0.735. Determine the concentration of the sample. (2 marks)

QUESTION TWO (20 MARKS)

- a) Briefly explain why atomic emission is more sensitive to flame instability than atomic absorption. (2 marks)
- b) Discuss the following types of interferences in atomic spectroscopy and discuss the methods to compensate for each interference.
- Matrix interference (5 marks)
 - Ionization interferences (4 marks)
- c) Describe the basic difference between atomic emission and atomic absorption spectroscopy. (2 marks)
- d) Discuss the atomization process in atomic absorption spectroscopy with specific reference to the analysis of an aqueous MgCl_2 solution. (5 marks)
- e) Briefly explain the potential advantages of derivatization of analytes prior to gas chromatography (GC) analysis. (2 marks)

QUESTION THREE (20 MARKS)

- a) In the analysis of some analgesic preparations using a polar stationary phase and a mobile phase composed of 10% water and 90% methanol, four compounds were seen to elute in the following order: caffeine, acetaminophen, aspirin, ibuprofen. The last two compounds were widely separated by several minutes. Briefly explain how the time required for the experiment can be decreased while maintaining adequate resolution. (2 marks)
- b) The following data was obtained during liquid chromatography analysis of acetaminophen, a headache medicine. The column was a C_{18} , the mobile phase was 50:50 methanol: water, and the detector was UV-Vis spectrometer (output measured in absorbance). The detector response for five different standards and a sample is presented in the following table.

| Concentration (ppm) | Absorbance |
|---------------------|------------|
| 1.00 | 0.098 |
| 2.00 | 0.211 |
| 4.00 | 0.402 |
| 8.00 | 0.699 |
| 10.00 | 0.881 |
| Sample | 0.296 |

- Calculate the slope and intercept of the regression line. (5 marks)
 - Determine the equation of the regression line for the above data. (1 mark)
 - Determine the concentration of acetaminophen in the sample. (1 mark)
- c) Explain why a thermal conductivity detector respond to all analytes except the carrier gas while flame ionization detector is not a universal detector. (4 marks)
- d) Acetone shows UV absorption maxima at λ_{max} 280 nm ($\epsilon_{\text{max}} = 15$) and λ_{max} 190 nm ($\epsilon_{\text{max}} = 100$).

- (i) Identify the electronic transition for each. (2 marks)
 (ii) Identify the absorption that is more intense. (1 mark)
- e) Sodium vapor lamps are commonly used to illuminate highways because of their intense yellow-orange emissions at 589 nm.
- (i) Calculate the energy, in joules, of one photon of this light. (2 marks)
 (ii) Calculate the energy, in kilojoules, of one mole of such photons. (2 marks)

QUESTION FOUR (20 MARKS)

- a) Compounds E and F were separated on a 1.0 m column. Symmetrical peaks were obtained and the retention times and peak widths at the base are shown below:

| | Retention time | Peak width at base/min |
|-----------------------|----------------|------------------------|
| Non-retained compound | 1.4 | - |
| Compound E | 16.0 | 1.07 |
| Compound F | 17.1 | 1.10 |

Calculate;

- (i) The capacity factor and selectivity factor for compounds E and F. (4 marks)
 (ii) The resolution between the peaks. (2 mark)
 (iii) The number of theoretical plates for the compounds E and F (2 ½ marks)
 (iv) The plate height in mm for compounds E and F (1 ½ mark)
- b) Briefly explain why internal standard method is the most appropriate in flame emission methods where variations in aspiration rate are expected. (2 marks)
- c) In a series of experiments on the determination of tin in foodstuffs, samples were boiled with hydrochloric acid under reflux for different times. Some of the results are shown below:

| Refluxing Time (Min) | Tin Found (mgkg ⁻¹) |
|----------------------|---------------------------------|
| 30 | 55, 57, 59, 56, 56, 59 |
| 75 | 57, 55, 58, 59, 59, 59 |

Test whether the mean amount of tin found differ significantly for the two boiling times.

(8 marks)

STATISTICAL TABLES

Table A.1 $F(z)$, the standard normal cumulative distribution function

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| -3.4 | 0.0003 | 0.0003 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0005 | 0.0005 |
| -3.3 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0007 |
| -3.2 | 0.0007 | 0.0007 | 0.0007 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0009 | 0.0009 | 0.0009 |
| -3.1 | 0.0010 | 0.0010 | 0.0010 | 0.0011 | 0.0011 | 0.0011 | 0.0012 | 0.0012 | 0.0013 | 0.0013 |
| -3.0 | 0.0013 | 0.0014 | 0.0014 | 0.0015 | 0.0015 | 0.0016 | 0.0016 | 0.0017 | 0.0018 | 0.0018 |
| -2.9 | 0.0019 | 0.0019 | 0.0020 | 0.0021 | 0.0021 | 0.0022 | 0.0023 | 0.0023 | 0.0024 | 0.0025 |
| -2.8 | 0.0026 | 0.0026 | 0.0027 | 0.0028 | 0.0029 | 0.0030 | 0.0031 | 0.0032 | 0.0033 | 0.0034 |
| -2.7 | 0.0035 | 0.0036 | 0.0037 | 0.0038 | 0.0039 | 0.0040 | 0.0041 | 0.0043 | 0.0044 | 0.0045 |
| -2.6 | 0.0047 | 0.0048 | 0.0049 | 0.0051 | 0.0052 | 0.0054 | 0.0055 | 0.0057 | 0.0059 | 0.0060 |
| -2.5 | 0.0062 | 0.0064 | 0.0066 | 0.0068 | 0.0069 | 0.0071 | 0.0073 | 0.0075 | 0.0078 | 0.0080 |
| -2.4 | 0.0082 | 0.0084 | 0.0087 | 0.0089 | 0.0091 | 0.0094 | 0.0096 | 0.0099 | 0.0102 | 0.0104 |
| -2.3 | 0.0107 | 0.0110 | 0.0113 | 0.0116 | 0.0119 | 0.0122 | 0.0125 | 0.0129 | 0.0132 | 0.0136 |
| -2.2 | 0.0139 | 0.0143 | 0.0146 | 0.0150 | 0.0154 | 0.0158 | 0.0162 | 0.0166 | 0.0170 | 0.0174 |
| -2.1 | 0.0179 | 0.0183 | 0.0188 | 0.0192 | 0.0197 | 0.0202 | 0.0207 | 0.0212 | 0.0217 | 0.0222 |
| -2.0 | 0.0228 | 0.0233 | 0.0239 | 0.0244 | 0.0250 | 0.0256 | 0.0262 | 0.0268 | 0.0274 | 0.0281 |
| -1.9 | 0.0287 | 0.0294 | 0.0301 | 0.0307 | 0.0314 | 0.0322 | 0.0329 | 0.0336 | 0.0344 | 0.0351 |
| -1.8 | 0.0359 | 0.0367 | 0.0375 | 0.0384 | 0.0392 | 0.0401 | 0.0409 | 0.0418 | 0.0427 | 0.0436 |
| -1.7 | 0.0446 | 0.0455 | 0.0465 | 0.0475 | 0.0485 | 0.0495 | 0.0505 | 0.0516 | 0.0526 | 0.0537 |
| -1.6 | 0.0548 | 0.0559 | 0.0571 | 0.0582 | 0.0594 | 0.0606 | 0.0618 | 0.0630 | 0.0643 | 0.0655 |
| -1.5 | 0.0668 | 0.0681 | 0.0694 | 0.0708 | 0.0721 | 0.0735 | 0.0749 | 0.0764 | 0.0778 | 0.0793 |

Table A.1 Continued

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| -1.4 | 0.0808 | 0.0823 | 0.0838 | 0.0853 | 0.0869 | 0.0885 | 0.0901 | 0.0918 | 0.0934 | 0.0951 |
| -1.3 | 0.0968 | 0.0985 | 0.1003 | 0.1020 | 0.1038 | 0.1056 | 0.1075 | 0.1093 | 0.1112 | 0.1131 |
| -1.2 | 0.1151 | 0.1170 | 0.1190 | 0.1210 | 0.1230 | 0.1251 | 0.1271 | 0.1292 | 0.1314 | 0.1335 |
| -1.1 | 0.1357 | 0.1379 | 0.1401 | 0.1423 | 0.1446 | 0.1469 | 0.1492 | 0.1515 | 0.1539 | 0.1562 |
| -1.0 | 0.1587 | 0.1611 | 0.1635 | 0.1660 | 0.1685 | 0.1711 | 0.1736 | 0.1762 | 0.1788 | 0.1814 |
| -0.9 | 0.1841 | 0.1867 | 0.1894 | 0.1922 | 0.1949 | 0.1977 | 0.2005 | 0.2033 | 0.2061 | 0.2090 |
| -0.8 | 0.2119 | 0.2148 | 0.2177 | 0.2206 | 0.2236 | 0.2266 | 0.2296 | 0.2327 | 0.2358 | 0.2389 |
| -0.7 | 0.2420 | 0.2451 | 0.2483 | 0.2514 | 0.2546 | 0.2578 | 0.2611 | 0.2643 | 0.2676 | 0.2709 |
| -0.6 | 0.2743 | 0.2776 | 0.2810 | 0.2843 | 0.2877 | 0.2912 | 0.2946 | 0.2981 | 0.3015 | 0.3050 |
| -0.5 | 0.3085 | 0.3121 | 0.3156 | 0.3192 | 0.3228 | 0.3264 | 0.3300 | 0.3336 | 0.3372 | 0.3409 |
| -0.4 | 0.3446 | 0.3483 | 0.3520 | 0.3557 | 0.3594 | 0.3632 | 0.3669 | 0.3707 | 0.3745 | 0.3783 |
| -0.3 | 0.3821 | 0.3859 | 0.3897 | 0.3936 | 0.3974 | 0.4013 | 0.4052 | 0.4090 | 0.4129 | 0.4168 |
| -0.2 | 0.4207 | 0.4247 | 0.4286 | 0.4325 | 0.4364 | 0.4404 | 0.4443 | 0.4483 | 0.4522 | 0.4562 |
| -0.1 | 0.4602 | 0.4641 | 0.4681 | 0.4721 | 0.4761 | 0.4801 | 0.4840 | 0.4880 | 0.4920 | 0.4960 |
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6579 |
| 0.5 | 0.6915 | 0.6950 | 0.6965 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |

Table A.2 The *t*-distribution

| Value of <i>t</i> for a confidence interval of Critical value of $ t $ for <i>P</i> values of number of degrees of freedom | 90% 0.10 | 95% 0.05 | 98% 0.02 | 99% 0.01 |
|--|-------------|-------------|-------------|-------------|
| 1 | 6.31 | 12.71 | 31.82 | 63.66 |
| 2 | 2.92 | 4.30 | 6.96 | 9.92 |
| 3 | 2.35 | 3.18 | 4.54 | 5.84 |
| 4 | 2.13 | 2.78 | 3.75 | 4.60 |
| 5 | 2.02 | 2.57 | 3.36 | 4.03 |
| 6 | 1.94 | 2.45 | 3.14 | 3.71 |
| 7 | 1.89 | 2.36 | 3.00 | 3.50 |
| 8 | 1.86 | 2.31 | 2.90 | 3.36 |
| 9 | 1.83 | 2.26 | 2.82 | 3.25 |
| 10 | 1.81 | 2.23 | 2.76 | 3.17 |
| 12 | 1.78 | 2.18 | 2.68 | 3.05 |
| 14 | 1.76 | 2.14 | 2.62 | 2.98 |
| 16 | 1.75 | 2.12 | 2.58 | 2.92 |
| 18 | 1.73 | 2.10 | 2.55 | 2.88 |
| 20 | 1.72 | 2.09 | 2.53 | 2.85 |
| 30 | 1.70 | 2.04 | 2.46 | 2.75 |
| 50 | 1.68 | 2.01 | 2.40 | 2.68 |
| ∞ | 1.64 | 1.96 | 2.33 | 2.58 |

The critical values of $|t|$ are appropriate for a *two-tailed* test. For a *one-tailed* test the value is taken from the column for *twice* the desired *P*-value, e.g. for a one-tailed test, $P = 0.05$, 5 degrees of freedom, the critical value is read from the $P = 0.10$ column and is equal to 2.02.

Table A.3 Critical values of F for a one-tailed test ($P = 0.05$)

| v_2 | v_1 | | | | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 | 20 |
| 1 | 161.4 | 199.5 | 215.7 | 224.6 | 230.2 | 234.0 | 236.8 | 238.9 | 240.5 | 241.9 | 243.9 | 245.9 | 248.0 |
| 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 | 19.43 | 19.45 |
| 3 | 10.13 | 9.552 | 9.277 | 9.117 | 9.013 | 8.941 | 8.887 | 8.845 | 8.812 | 8.786 | 8.745 | 8.703 | 8.660 |
| 4 | 7.709 | 6.944 | 6.591 | 6.388 | 6.256 | 6.163 | 6.094 | 6.041 | 5.999 | 5.964 | 5.912 | 5.858 | 5.803 |
| 5 | 6.608 | 5.786 | 5.409 | 5.192 | 5.050 | 4.950 | 4.876 | 4.818 | 4.772 | 4.735 | 4.678 | 4.619 | 4.558 |
| 6 | 5.987 | 5.143 | 4.757 | 4.534 | 4.387 | 4.284 | 4.207 | 4.147 | 4.099 | 4.060 | 4.000 | 3.938 | 3.874 |
| 7 | 5.591 | 4.737 | 4.347 | 4.120 | 3.972 | 3.866 | 3.787 | 3.726 | 3.677 | 3.637 | 3.575 | 3.511 | 3.445 |
| 8 | 5.318 | 4.459 | 4.066 | 3.838 | 3.687 | 3.581 | 3.500 | 3.438 | 3.388 | 3.347 | 3.284 | 3.218 | 3.150 |
| 9 | 5.117 | 4.256 | 3.863 | 3.633 | 3.482 | 3.374 | 3.293 | 3.230 | 3.179 | 3.137 | 3.073 | 3.006 | 2.936 |
| 10 | 4.965 | 4.103 | 3.708 | 3.478 | 3.326 | 3.217 | 3.135 | 3.072 | 3.020 | 2.978 | 2.913 | 2.845 | 2.774 |
| 11 | 4.844 | 3.982 | 3.587 | 3.357 | 3.204 | 3.095 | 3.012 | 2.948 | 2.896 | 2.854 | 2.788 | 2.719 | 2.646 |
| 12 | 4.747 | 3.885 | 3.490 | 3.259 | 3.106 | 2.996 | 2.913 | 2.849 | 2.796 | 2.753 | 2.687 | 2.617 | 2.544 |
| 13 | 4.667 | 3.806 | 3.411 | 3.179 | 3.025 | 2.915 | 2.832 | 2.767 | 2.714 | 2.671 | 2.604 | 2.533 | 2.459 |
| 14 | 4.600 | 3.739 | 3.344 | 3.112 | 2.958 | 2.848 | 2.764 | 2.699 | 2.646 | 2.602 | 2.534 | 2.463 | 2.388 |
| 15 | 4.543 | 3.682 | 3.287 | 3.056 | 2.901 | 2.790 | 2.707 | 2.641 | 2.588 | 2.544 | 2.475 | 2.403 | 2.328 |
| 16 | 4.494 | 3.634 | 3.239 | 3.007 | 2.852 | 2.741 | 2.657 | 2.591 | 2.538 | 2.494 | 2.425 | 2.352 | 2.276 |
| 17 | 4.451 | 3.592 | 3.197 | 2.965 | 2.810 | 2.699 | 2.614 | 2.548 | 2.494 | 2.450 | 2.381 | 2.308 | 2.230 |
| 18 | 4.414 | 3.555 | 3.160 | 2.928 | 2.773 | 2.661 | 2.577 | 2.510 | 2.456 | 2.412 | 2.342 | 2.269 | 2.191 |
| 19 | 4.381 | 3.522 | 3.127 | 2.895 | 2.740 | 2.628 | 2.544 | 2.477 | 2.423 | 2.378 | 2.308 | 2.234 | 2.155 |
| 20 | 4.351 | 3.493 | 3.098 | 2.866 | 2.711 | 2.599 | 2.514 | 2.447 | 2.393 | 2.348 | 2.278 | 2.203 | 2.124 |

v_1 = number of degrees of freedom of the numerator; v_2 = number of degrees of freedom of the denominator.

Table A.4 Critical values of F for a two-tailed test ($P = 0.05$)

| v_2 | v_1 | | | | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 | 20 |
| 1 | 647.8 | 799.5 | 864.2 | 899.6 | 921.8 | 937.1 | 948.2 | 956.7 | 963.3 | 968.6 | 976.7 | 984.9 | 993.1 |
| 2 | 38.51 | 39.00 | 39.17 | 39.25 | 39.30 | 39.33 | 39.36 | 39.37 | 39.39 | 39.40 | 39.41 | 39.43 | 39.45 |
| 3 | 17.44 | 16.04 | 15.44 | 15.10 | 14.88 | 14.73 | 14.62 | 14.54 | 14.47 | 14.42 | 14.34 | 14.25 | 14.17 |
| 4 | 12.22 | 10.65 | 9.979 | 9.605 | 9.364 | 9.197 | 9.074 | 8.980 | 8.905 | 8.844 | 8.751 | 8.657 | 8.560 |
| 5 | 10.01 | 8.434 | 7.764 | 7.388 | 7.146 | 6.978 | 6.853 | 6.757 | 6.681 | 6.619 | 6.525 | 6.428 | 6.329 |
| 6 | 8.813 | 7.260 | 6.599 | 6.227 | 5.988 | 5.820 | 5.695 | 5.600 | 5.523 | 5.461 | 5.366 | 5.269 | 5.168 |
| 7 | 8.073 | 6.542 | 5.890 | 5.523 | 5.285 | 5.119 | 4.995 | 4.899 | 4.823 | 4.761 | 4.666 | 4.568 | 4.467 |
| 8 | 7.571 | 6.059 | 5.416 | 5.053 | 4.817 | 4.652 | 4.529 | 4.433 | 4.357 | 4.295 | 4.200 | 4.101 | 3.999 |
| 9 | 7.209 | 5.715 | 5.078 | 4.718 | 4.484 | 4.320 | 4.197 | 4.102 | 4.026 | 3.964 | 3.868 | 3.769 | 3.667 |
| 10 | 6.937 | 5.456 | 4.826 | 4.468 | 4.236 | 4.072 | 3.950 | 3.855 | 3.779 | 3.717 | 3.621 | 3.522 | 3.419 |
| 11 | 6.724 | 5.256 | 4.630 | 4.275 | 4.044 | 3.881 | 3.759 | 3.664 | 3.588 | 3.526 | 3.430 | 3.330 | 3.226 |
| 12 | 6.554 | 5.096 | 4.474 | 4.121 | 3.891 | 3.728 | 3.607 | 3.512 | 3.436 | 3.374 | 3.277 | 3.177 | 3.073 |
| 13 | 6.414 | 4.965 | 4.347 | 3.996 | 3.767 | 3.604 | 3.483 | 3.388 | 3.312 | 3.250 | 3.153 | 3.053 | 2.948 |
| 14 | 6.298 | 4.857 | 4.242 | 3.892 | 3.663 | 3.501 | 3.380 | 3.285 | 3.209 | 3.147 | 3.050 | 2.949 | 2.844 |
| 15 | 6.200 | 4.765 | 4.153 | 3.804 | 3.576 | 3.415 | 3.293 | 3.199 | 3.123 | 3.060 | 2.963 | 2.862 | 2.756 |
| 16 | 6.115 | 4.687 | 4.077 | 3.729 | 3.502 | 3.341 | 3.219 | 3.125 | 3.049 | 2.986 | 2.889 | 2.788 | 2.681 |
| 17 | 6.042 | 4.619 | 4.011 | 3.665 | 3.438 | 3.277 | 3.156 | 3.061 | 2.985 | 2.922 | 2.825 | 2.723 | 2.616 |
| 18 | 5.978 | 4.560 | 3.954 | 3.608 | 3.382 | 3.221 | 3.100 | 3.005 | 2.929 | 2.866 | 2.769 | 2.667 | 2.559 |
| 19 | 5.922 | 4.508 | 3.903 | 3.559 | 3.333 | 3.172 | 3.051 | 2.956 | 2.880 | 2.817 | 2.720 | 2.617 | 2.509 |
| 20 | 5.871 | 4.461 | 3.859 | 3.515 | 3.289 | 3.128 | 3.007 | 2.913 | 2.837 | 2.774 | 2.676 | 2.573 | 2.464 |

v_1 = number of degrees of freedom of the numerator; v_2 = number of degrees of freedom of the denominator.