

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN COMPUTER SCIENCE

COSC 102: DISCRETE STRUCTURES

STREAMS: BSC. COMP SCI

TIME: 2 HOURS

DAY/DATE: WEDNESDAY 16/04/2025

11.30 A.M. – 1.30 P.M.

**INSTRUCTIONS:**

1. Answer **Question ONE (Compulsory)** and **any other TWO questions**.
2. Diagrams and illustrations should be used where necessary.
3. This is a **closed book** examination; reference materials and mobile phones are not allowed.
4. Write clearly and use your time wisely.

**SECTION A: COMPULSORY [attempt all questions in this section]**

**QUESTION ONE (30 MARKS)**

a) Define the following terms and give an example for each:

- i) **Set** [2 marks]
- ii) **Proposition** [2 marks]
- iii) **Subset** [2 marks]

b) Identify whether the following statements are **proposition**, and explain why:

- i) "The sun rises in the west." [2 marks]
- ii) " $3 + 4 = 7$ ." [2 marks]

c) Consider the sets:

$$A = \{1, 2, 3, 4, 5\}, B = \{3, 4, 5, 6, 7\}$$

Find:

- i)  $A \cup B$  [3 marks]
- ii)  $A \cap B$  [3 marks]
- iii)  $A - B$  [3 marks]

d) Express the **negation** of the following statements without using the word "not":

- i) "Today is a holiday." [2 marks]
- ii) "All students passed the exam." [2 marks]

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- e) Suppose there are **50 people in a room**. Show that at least **one month must have at least five people** with the same birth month. [3 marks]

### SECTION B: ANSWER ANY TWO QUESTIONS

#### QUESTION TWO (20 MARKS)

- a) Prove, using **direct proof**, that the square of any even natural number is also even. [6 marks]
- b) Prove, using **contradiction**, that the square of any odd natural number is also odd. [6 marks]
- c) Suppose variable names in a given programming language can be either a single uppercase letter or an uppercase letter followed by a digit, find the number of possible variable names [4 marks]
- d) How many bit strings of length 8 either start with a 1 or end with two bits 00? [4 marks]

#### QUESTION THREE (20 MARKS)

- a) Construct the truth table for the logical expression:  
 $(P \wedge Q) \vee (\neg P \wedge R) \wedge (P \wedge Q) \vee (\neg P \wedge R)$ . [6 marks]
- b) A computer system requires that passwords be between **six and eight characters long**, where each character is an **uppercase letter or a digit**. Each password must contain at least **one digit**. Find the number of valid passwords. [8 marks]
- c) Find the number of ways to arrange six objects {**A, B, C, D, E, F**} taken three at a time. [6 marks]

#### QUESTION FOUR (20 MARKS)

- a) Convert the following statement into symbolic logic and determine its **Contrapositive, Converse, and Inverse**:  
*"If it is raining, then the home team wins."* [8 marks]
- b) Consider the **Island of Questioners**, where every inhabitant belongs to one of two types:  
**Type A** only asks questions where the answer is "Yes".  
**Type B** only asks questions where the answer is "No".  
Muchiri and Chebet are two inhabitants of the island. One day, Muchiri asks, *"Are Chebet and I both of type B?"*  
Determine the type of Chebet, with explanation. [8 marks]
- (c) A committee of 3 students needs to be formed from a group of 5 students. How many different committees can be formed? [6 marks]

**QUESTION FIVE (20 MARKS)**

- (a) The difference of A and B, is the set of all elements that belong to A but not to B. Use Venn diagram to demonstrate this difference. [6 marks]
- (b) For each of the sets A and B below, find  $A \cup B$  and  $A \cap B$  [8 marks]
- (i)  $A = \{3, 2, a\}$ ,  $B = \{2, 3, a\}$
- (ii)  $A = \{4, 7, -1\}$ ,  $B = \{7, 3, 4\}$
- (c) Given the sets  $A = \{1, 2, 3\}$  and  $B = \{a, b, c\}$ , define a function  $f: A \rightarrow B$  that is injective but not surjective. [6 marks]
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