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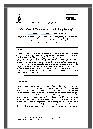
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Transitivity Action of the Cartesian Product of the Alternating Group Acting on a Cartesian Product of Ordered Sets of Triples

Maraka K. Moses ^{a*}, Musundi W. Sammy ^a and Lewis N. Nyaga ^b

^a Department of Physical Sciences, Chuka University; P.O. Box 109-60400 Chuka, Kenya.

^b Department of Pure and Applied Mathematics, Jomo Kenyatta University of Agriculture and Technology, P.O. Box 62000-00200 Nairobi, Kenya.

swmusundi@chuka.ac.ke

Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

In this paper, we investigate some transitivity action properties of the cartesian product of the alternating group $A_n (n \geq 5)$ acting on a cartesian product of ordered sets of triples using the Orbit-Stabilizer Theorem by showing that the length of the orbit (p, s, v) in $A_n \times A_n \times A_n$, $(n \geq 5)$ acting on $P^{[3]} \times S^{[3]} \times V^{[3]}$ is equivalent to the cardinality of $P^{[3]} \times S^{[3]} \times V^{[3]}$ to imply transitivity.

Keywords: Orbits; stabilizer; transitivity action; ordered sets of triples; cartesian product; fixed point.

1 Preliminaries

1.1 Notation and Terminology

In this paper, we shall represent the following notations as: \sum - sum over i ; A_n -an alternating group of degree n and order $\frac{n!}{2}$; $|G|$ - the order of a group G ; $|G:H|$ -Index of H in G ; $P^{[3]}$ - the set of an ordered triple from set $P = \{1,2,3,\dots,n\}$; $S^{[3]}$ - the set of an ordered triple from set $S = \{n+1, n+2,\dots,2n\}$; $V^{[3]}$ - the set of an

*Corresponding author: Email: swmusundi@chuka.ac.ke;

ordered triple from set $V = \{2n + 1, 2n + 2, \dots, 3n\}$; $[a, b, c]$ -Ordered triple; $A_n \times A_n \times A_n$ -Cartesian product of alternating group A_n ; $P^{[3]} \times S^{[3]} \times V^{[3]}$ -Cartesian product of ordered sets of triples $P^{[3]}$, $S^{[3]}$ and $V^{[3]}$.

Definition 1.1.1. Group action [1]: Let P be a non-empty set. A group G is said to act on the left of P if for each $g \in G$ and each $p \in P$ there corresponds a unique element $gp \in P$ such that:

- (i) $(g_1g_2)p = g_1(g_2p)$, $g_1, g_2 \in G$ and $p \in P$.
- (ii) For any $p \in P$, $ep = p$, where e is the identity in G .

The action of G from the right on P can be defined in the same manner.

Definition 1.1.2. Orbit [2]: The action of a group G on a set P partitions P into disjoint equivalence classes referred to as orbits or transitivity classes of action. The orbit containing $p \in P$ is denoted by $Orb_G(p)$.

Definition 1.1.3. Stabilizer of an element [3]: Let G act on a set P and $p \in P$. The stabilizer of p in G , denoted by $Stab_G(p)$ is given by $Stab_G(p) = \{g \in G | gp = p\}$.

Definition 1.1.4. Fixed point [1]: Let G act on a set P . The set of elements of P fixed by $g \in G$ is called the fixed-point set of G and is denoted by $Fix(g)$. Thus $Fix(g) = \{p \in P | gp = p\}$.

Definition 1.1.5. Transitive group [4]: If the action of a group G on set P has only one orbit, then we say that G acts transitively on P . In other words, G acts transitively on P if for every pair of points $p, s \in P$, there exists $g \in G$ such that $gp = s$.

Definition 1.1.6. Conjugate group [2]: A group G with two subgroups H and K , then they are said to be conjugate if $H = gkg^{-1}$ for some $g \in G$.

Theorem 1.1.7 [5]: Two permutations in A_n are conjugate if and only if, they have the same cycle type and if $g \in S_n$ has cycle type $(\alpha_1, \alpha_2, \dots, \alpha_n)$, then the number of permutations in S_n conjugate to g is, $\frac{n!}{\prod_{i=1}^n i^{\alpha_i} \alpha_i!}$.

Theorem 1.1.8 (Orbit – Stabilizer Theorem, [3, p.72]): Let G act on a set P . Then $|Orb_G(p)| = |G : Stab_G(p)|$.

Theorem 1.1.9 [3]: Let G be a group acting on a finite set P . Then the number of G -orbits in P is,

$$\frac{1}{|G|} \sum_{g \in G} |Fix(g)|.$$

Definition 1.1.10 (Direct product action, [4]): Let (G_1, P_1) and (G_2, P_2) be permutation groups. The direct product $G_1 \times G_2$ acts on the disjoint union $P_1 \cup P_2$ by the rule $p(g_1, g_2) = \begin{cases} pg_1, & \text{if } p \in P_1, \\ pg_2 & \text{if } p \in P_2 \end{cases}$ and on the Cartesian product $P_1 \times P_2$ by the rule $(p_1, p_2)(g_1, g_2) = (p_1g_1, p_2g_2)$.

Theorem 1.1.11 [6]: The $G_1 \times G_2 \times G_3$ -orbit containing $(p, s, v) \in P \times S \times V$ is given by $Orb_{G_1}(p) \times Orb_{G_2}(s) \times Orb_{G_3}(v)$ and the stabilizer of (p, s, v) is given by $Stab_{G_1}(p) \times Stab_{G_2}(s) \times Stab_{G_3}(v)$.

2 Introduction

Higman [7] introduced the rank of a group on finite permutation groups of rank 3. In 1970, Higman proved that the rank of the symmetric group S_n acting on 2-element subsets from the set $P = \{1, 2, \dots, n\}$ is 3 and the subdegrees are: $1, 2(n - 1)$ and $\binom{n-2}{2}$. Cameron [4] worked on the suborbits of multiply transitive permutations and later in 1974 studied the suborbits of primitive groups.

Ndarinyo et al. [8] showed that the alternating group $A_n = 5,6,7$ acts transitively on unordered and ordered triples from the set $P = 1,2, \dots, n$ when $n \leq 7$ through determination of the number of orbits. Nyaga [9] proved that the direct product action of the alternating group on the Cartesian product of three sets is transitive. The ranks and subdegrees associated with this action for $n \geq 4$ is 8; and $1, (n - 1), (n - 1)^2, (n - 1)^3$ respectively. Muriuki et al. [10] showed that for the action of direct product of three symmetric groups on Cartesian product of three sets, the action is both transitive and imprimitive for all $n \geq 2$ and the associated rank is 2^3 . Mutua et al. [11] showed that the direct product of $S_n \times A_n$ on $X \times Y$ has its action both transitive and imprimitive when $n \geq 3$. The associated rank for this action is 6 when $n = 3$, but is 4 for all $n \geq 3$. Based on these results we investigate some properties of $A_n \times A_n \times A_n$, the cartesian product action of the alternating group acting on $P^{[3]} \times S^{[3]} \times V^{[3]}$, the cartesian product of ordered sets of triples.

The cartesian product of alternating group $A_n \times A_n \times A_n$, acts on $P^{[3]} \times S^{[3]} \times V^{[3]}$, by the rule;

$$g_1\{([1,2,3], [1,2,4], \dots, [n, n - 1, n - 3], [n, n - 1, n - 2])\} \times g_2\{([n + 1, n + 2, n + 3], [n + 1, n + 2, n + 4], \dots, [2n, 2n - 1, 2n - 3], [2n, 2n - 1, 2n - 2])\} \times g_3\{([2n + 1, 2n + 2, 2n + 3], [2n + 1, 2n + 2, 2n + 4], \dots, [3n, 3n - 1, 3n - 3], [3n, 3n - 1, 3n - 2])\} = \{g_1([1,2,3], [1,2,4], \dots, [n, n - 1, n - 3], [n, n - 1, n - 2]), g_2([n + 1, n + 2, n + 3], [n + 1, n + 2, n + 4], \dots, [2n, 2n - 1, 2n - 3], [2n, 2n - 1, 2n - 2]), g_3([2n + 1, 2n + 2, 2n + 3], [2n + 1, 2n + 2, 2n + 4], \dots, [3n, 3n - 1, 3n - 3], [3n, 3n - 1, 3n - 2])\};$$

$$\forall g_1, g_2, g_3 \in A_n \{([1,2,3], [1,2,4], \dots, [n, n - 1, n - 3], [n, n - 1, n - 2])\} \in P^{[3]}, \text{ set of ordered triples from the set } P = \{1,2,3, \dots, n\}; \{([n + 1, n + 2, n + 3], [n + 1, n + 2, n + 4], \dots, [2n, 2n - 1, 2n - 3], [2n, 2n - 1, 2n - 2])\} \in S^{[3]}, \text{ set of ordered triples from the set } S = \{n + 1, n + 2, \dots, 2n\}; \text{ and } \{([2n + 1, 2n + 2, 2n + 3], [2n + 1, 2n + 2, 2n + 4], \dots, [3n, 3n - 1, 3n - 3], [3n, 3n - 1, 3n - 2])\} \in V^{[3]}, \text{ set of ordered triples from the set } V = \{2n + 1, 2n + 2, \dots, 3n\}.$$

3 Main Results

Lemma 2.1: The action of $A_5 \times A_5 \times A_5$ on $P^{[3]} \times S^{[3]} \times V^{[3]}$ is transitive.

Proof: Let $G = A_5 \times A_5 \times A_5$ act on $P^{[3]} \times S^{[3]} \times V^{[3]}$ where; $\text{gap} > \text{Arrangements}([1,2,3,4,5],3)$; $P^{[3]} = \{[1, 2, 3], [1, 2, 4], [1, 2, 5], [1, 3, 2], [1, 3, 4], [1, 3, 5], [1, 4, 2], [1, 4, 3], [1, 4, 5], [1, 5, 2], [1, 5, 3], [1, 5, 4], [2, 1, 3], [2, 1, 4], [2, 1, 5], [2, 3, 1], [2, 3, 4], [2, 3, 5], [2, 4, 1], [2, 4, 3], [2, 4, 5], [2, 5, 1], [2, 5, 3], [2, 5, 4], [3, 1, 2], [3, 1, 4], [3, 1, 5], [3, 2, 1], [3, 2, 4], [3, 2, 5], [3, 4, 1], [3, 4, 2], [3, 4, 5], [3, 5, 1], [3, 5, 2], [3, 5, 4], [4, 1, 2], [4, 1, 3], [4, 1, 5], [4, 2, 1], [4, 2, 3], [4, 2, 5], [4, 3, 1], [4, 3, 2], [4, 3, 5], [4, 5, 1], [4, 5, 2], [4, 5, 3], [5, 1, 2], [5, 1, 3], [5, 1, 4], [5, 2, 1], [5, 2, 3], [5, 2, 4], [5, 3, 1], [5, 3, 2], [5, 3, 4], [5, 4, 1], [5, 4, 2], [5, 4, 3]\}$;
 $\text{gap} > \text{Arrangements}([6,7,8,9,10],3)$; $S^{[3]} = \{[6, 7, 8], [6, 7, 9], [6, 7, 10], [6, 8, 7], [6, 8, 9], [6, 8, 10], [6, 9, 7], [6, 9, 8], [6, 9, 10], [6, 10, 7], [6, 10, 8], [6, 10, 9], [7, 6, 8], [7, 6, 9], [7, 6, 10], [7, 8, 6], [7, 8, 9], [7, 8, 10], [7, 9, 6], [7, 9, 8], [7, 9, 10], [7, 10, 6], [7, 10, 8], [7, 10, 9], [8, 6, 7], [8, 6, 9], [8, 6, 10], [8, 7, 6], [8, 7, 9], [8, 7, 10], [8, 9, 6], [8, 9, 7], [8, 9, 10], [8, 10, 6], [8, 10, 7], [8, 10, 9], [9, 6, 7], [9, 6, 8], [9, 6, 10], [9, 7, 6], [9, 7, 8], [9, 7, 10], [9, 8, 6], [9, 8, 7], [9, 8, 10], [9, 10, 6], [9, 10, 7], [9, 10, 8], [10, 6, 7], [10, 6, 8], [10, 6, 9], [10, 7, 6], [10, 7, 8], [10, 7, 9], [10, 8, 6], [10, 8, 7], [10, 8, 9], [10, 9, 6], [10, 9, 7], [10, 9, 8]\}$; and
 $\text{gap} > \text{Arrangements}([11,12,13,14,15],3)$; $V^{[3]} = \{[11, 12, 13], [11, 12, 14], [11, 12, 15], [11, 13, 12], [11, 13, 14], [11, 13, 15], [11, 14, 12], [11, 14, 13], [11, 14, 15], [11, 15, 12], [11, 15, 13], [11, 15, 14], [12, 11, 13], [12, 11, 14], [12, 11, 15], [12, 13, 11], [12, 13, 14], [12, 13, 15], [12, 14, 11], [12, 14, 13], [12, 14, 15], [12, 15, 11], [12, 15, 13], [12, 15, 14], [13, 11, 12], [13, 11, 14], [13, 11, 15], [13, 12, 11], [13, 12, 14], [13, 12, 15], [13, 14, 11], [13, 14, 12], [13, 14, 15], [13, 15, 11], [13, 15, 12], [13, 15, 14], [14, 11, 12], [14, 11, 13], [14, 11, 15], [14, 12, 11], [14, 12, 13], [14, 12, 15], [14, 13, 11], [14, 13, 12], [14, 13, 15], [14, 15, 11], [14, 15, 12], [14, 15, 13], [15, 11, 12], [15, 11, 13], [15, 11, 14], [15, 12, 11], [15, 12, 13], [15, 12, 14], [15, 13, 11], [15, 13, 12], [15, 13, 14], [15, 14, 11], [15, 14, 12], [15, 14, 13]\}$.

The cartesian product of $P^{[3]} \times S^{[3]} \times V^{[3]}$ is generated using the GAP software with, $|P^{[3]} \times S^{[3]} \times V^{[3]}| = 216\,000$. G is generated by

$\langle \{(1\,2\,3\,4\,5), (123)\}, \{(6\,7\,8\,9\,10), (678)\}, \{(11\,12\,13\,14\,15), (11\,12\,13)\} \rangle$ using the GAP software. $([1,2,3], [6,7,8], [11,12,13])$ is fixed by an element $(g_p, g_s, g_v) \in G$ if and only if 1,2 and 3 comes from a single cycle in g_p ; 6,7 and 8 comes from a single cycle in g_s and 11,12 and 13 comes from a single cycle in g_v .

Therefore, $Stab_G([1,2,3], [6,7,8], [11,12,13]) = \{1,6,11\} = \{(e_p, e_s, e_v)\}$.
 $|Stab_G([1,2,3], [6,7,8], [11,12,13])| = 1$.

By Orbit-Stabilizer Theorem,

$$\begin{aligned} |Orb_G([1,2,3], [6,7,8], [11,12,13])| &= |G: Stab_G([1,2,3], [6,7,8], [11,12,13])| \\ &= \frac{|G|}{|Stab_G([1,2,3], [6,7,8], [11,12,13])|} \\ &= \frac{216000}{1} = 216000 = |P^{[3]} \times S^{[3]} \times V^{[3]}| \end{aligned}$$

Therefore, $A_5 \times A_5 \times A_5$ acts transitively on $P^{[3]} \times S^{[3]} \times V^{[3]}$.

Lemma 2.2: The action of $A_6 \times A_6 \times A_6$ on $P^{[3]} \times S^{[3]} \times V^{[3]}$ is transitive.

Proof: Let $G = A_6 \times A_6 \times A_6$ act on $P^{[3]} \times S^{[3]} \times V^{[3]}$ where;

gap> Arrangements([1,2,3,4,5,6],3); $P^{[3]} = [1, 2, 3], [1, 2, 4], [1, 2, 5], [1, 2, 6], [1, 3, 2], [1, 3, 4], [1, 3, 5], [1, 3, 6], [1, 4, 2], [1, 4, 3], [1, 4, 5], [1, 4, 6], [1, 5, 2], [1, 5, 3], [1, 5, 4], [1, 5, 6], [1, 6, 2], [1, 6, 3], [1, 6, 4], [1, 6, 5], [2, 1, 3], [2, 1, 4], [2, 1, 5], [2, 1, 6], [2, 3, 1], [2, 3, 4], [2, 3, 5], [2, 3, 6], [2, 4, 1], [2, 4, 3], [2, 4, 5], [2, 4, 6], [2, 5, 1], [2, 5, 3], [2, 5, 4], [2, 5, 6], [2, 6, 1], [2, 6, 3], [2, 6, 4], [2, 6, 5], [3, 1, 2], [3, 1, 4], [3, 1, 5], [3, 1, 6], [3, 2, 1], [3, 2, 4], [3, 2, 5], [3, 2, 6], [3, 4, 1], [3, 4, 2], [3, 4, 5], [3, 4, 6], [3, 5, 1], [3, 5, 2], [3, 5, 4], [3, 5, 6], [3, 6, 1], [3, 6, 2], [3, 6, 4], [3, 6, 5], [4, 1, 2], [4, 1, 3], [4, 1, 5], [4, 1, 6], [4, 2, 1], [4, 2, 3], [4, 2, 5], [4, 2, 6], [4, 3, 1], [4, 3, 2], [4, 3, 5], [4, 3, 6], [4, 5, 1], [4, 5, 2], [4, 5, 3], [4, 5, 6], [4, 6, 1], [4, 6, 2], [4, 6, 3], [4, 6, 5], [5, 1, 2], [5, 1, 3], [5, 1, 4], [5, 1, 6], [5, 2, 1], [5, 2, 3], [5, 2, 4], [5, 2, 6], [5, 3, 1], [5, 3, 2], [5, 3, 4], [5, 3, 6], [5, 4, 1], [5, 4, 2], [5, 4, 3], [5, 4, 6], [5, 6, 1], [5, 6, 2], [5, 6, 3], [5, 6, 4], [6, 1, 2], [6, 1, 3], [6, 1, 4], [6, 1, 5], [6, 2, 1], [6, 2, 3], [6, 2, 4], [6, 2, 5], [6, 3, 1], [6, 3, 2], [6, 3, 4], [6, 3, 5], [6, 4, 1], [6, 4, 2], [6, 4, 3], [6, 4, 5], [6, 5, 1], [6, 5, 2], [6, 5, 3], [6, 5, 4];$

gap> Arrangements([7,8,9,10,11,12],3); $S^{[3]} = [7, 8, 9], [7, 8, 10], [7, 8, 11], [7, 8, 12], [7, 9, 8], [7, 9, 10], [7, 9, 11], [7, 9, 12], [7, 10, 8], [7, 10, 9], [7, 10, 11], [7, 10, 12], [7, 11, 8], [7, 11, 9], [7, 11, 10], [7, 11, 12], [7, 12, 8], [7, 12, 9], [7, 12, 10], [7, 12, 11], [8, 7, 9], [8, 7, 10], [8, 7, 11], [8, 7, 12], [8, 9, 7], [8, 9, 10], [8, 9, 11], [8, 9, 12], [8, 10, 7], [8, 10, 9], [8, 10, 11], [8, 10, 12], [8, 11, 7], [8, 11, 9], [8, 11, 10], [8, 11, 12], [8, 12, 7], [8, 12, 9], [8, 12, 10], [8, 12, 11], [9, 7, 8], [9, 7, 10], [9, 7, 11], [9, 7, 12], [9, 8, 7], [9, 8, 10], [9, 8, 11], [9, 8, 12], [9, 10, 7], [9, 10, 8], [9, 10, 11], [9, 10, 12], [9, 11, 7], [9, 11, 8], [9, 11, 10], [9, 11, 12], [9, 12, 7], [9, 12, 8], [9, 12, 10], [9, 12, 11], [10, 7, 8], [10, 7, 9], [10, 7, 11], [10, 7, 12], [10, 8, 7], [10, 8, 9], [10, 8, 11], [10, 8, 12], [10, 9, 7], [10, 9, 8], [10, 9, 11], [10, 9, 12], [10, 11, 7], [10, 11, 8], [10, 11, 9], [10, 11, 12], [10, 12, 7], [10, 12, 8], [10, 12, 9], [10, 12, 11], [11, 7, 8], [11, 7, 9], [11, 7, 10], [11, 7, 12], [11, 8, 7], [11, 8, 9], [11, 8, 10], [11, 8, 12], [11, 9, 7], [11, 9, 8], [11, 9, 10], [11, 9, 12], [11, 10, 7], [11, 10, 8], [11, 10, 9], [11, 10, 12], [11, 12, 7], [11, 12, 8], [11, 12, 9], [11, 12, 10], [12, 7, 8], [12, 7, 9], [12, 7, 10], [12, 7, 11], [12, 8, 7], [12, 8, 9], [12, 8, 10], [12, 8, 11], [12, 9, 7], [12, 9, 8], [12, 9, 10], [12, 9, 11], [12, 9, 12], [12, 10, 7], [12, 10, 8], [12, 10, 9], [12, 10, 12], [12, 11, 7], [12, 11, 8], [12, 11, 9], [12, 11, 12], [12, 12, 7], [12, 12, 8], [12, 12, 9], [12, 12, 10], [12, 12, 11], [12, 12, 12];$

12, 8, 11], [12, 9, 7], [12, 9, 8], [12, 9, 10], [12, 9, 11], [12, 10, 7], [12, 10, 8], [12, 10, 9], [12, 10, 11], [12, 11, 7], [12, 11, 8], [12, 11, 9], [12, 11, 10]}

and;

gap> Arrangements([13,14,15,16,17,18],3); $V^{[3]} = \{ [13, 14, 15], [13, 14, 16], [13, 14, 17], [13, 14, 18], [13, 15, 14], [13, 15, 16], [13, 15, 17], [13, 15, 18], [13, 16, 14], [13, 16, 15], [13, 16, 17], [13, 16, 18], [13, 17, 14], [13, 17, 15], [13, 17, 16], [13, 17, 18], [13, 18, 14], [13, 18, 15], [13, 18, 16], [13, 18, 17], [14, 13, 15], [14, 13, 16], [14, 13, 17], [14, 13, 18], [14, 15, 13], [14, 15, 16], [14, 15, 17], [14, 15, 18], [14, 16, 13], [14, 16, 15], [14, 16, 17], [14, 16, 18], [14, 17, 13], [14, 17, 15], [14, 17, 16], [14, 17, 18], [14, 18, 13], [14, 18, 15], [14, 18, 16], [14, 18, 17], [15, 13, 14], [15, 13, 16], [15, 13, 17], [15, 13, 18], [15, 14, 13], [15, 14, 16], [15, 14, 17], [15, 14, 18], [15, 16, 13], [15, 16, 14], [15, 16, 17], [15, 16, 18], [15, 17, 13], [15, 17, 14], [15, 17, 16], [15, 17, 18], [15, 18, 13], [15, 18, 14], [15, 18, 16], [15, 18, 17], [16, 13, 14], [16, 13, 15], [16, 13, 17], [16, 13, 18], [16, 14, 13], [16, 14, 15], [16, 14, 17], [16, 14, 18], [16, 15, 13], [16, 15, 14], [16, 15, 17], [16, 15, 18], [16, 17, 13], [16, 17, 14], [16, 17, 15], [16, 17, 18], [16, 18, 13], [16, 18, 14], [16, 18, 15], [16, 18, 17], [17, 13, 14], [17, 13, 15], [17, 13, 16], [17, 13, 18], [17, 14, 13], [17, 14, 15], [17, 14, 16], [17, 14, 18], [17, 15, 13], [17, 15, 14], [17, 15, 16], [17, 15, 18], [17, 16, 13], [17, 16, 14], [17, 16, 15], [17, 16, 18], [17, 18, 13], [17, 18, 14], [17, 18, 15], [17, 18, 16], [18, 13, 14], [18, 13, 15], [18, 13, 16], [18, 13, 17], [18, 14, 13], [18, 14, 15], [18, 14, 16], [18, 14, 17], [18, 15, 13], [18, 15, 14], [18, 15, 16], [18, 15, 17], [18, 16, 13], [18, 16, 14], [18, 16, 15], [18, 16, 17], [18, 17, 13], [18, 17, 14], [18, 17, 15], [18, 17, 16] \}$.

The cartesian product of $P^{[3]} \times S^{[3]} \times V^{[3]}$ is generated using the GAP software with $|P^{[3]} \times S^{[3]} \times V^{[3]}| = 1728000$. G is generated by

$\langle \{(123456), (123)\}, \{(7\ 8\ 9\ 10\ 11\ 12), (789)\}, \{(13\ 14\ 15\ 16\ 17\ 18), (13\ 14\ 15)\} \rangle$ using the GAP software. $([1,2,3], [7,8,9], [13,14,15])$ is fixed by an element $(g_p, g_s, g_v) \in G$ if and only if 1,2 and 3 comes from a single cycle in g_p ; 7,8 and 9 comes from a single cycle in g_s and 13,14 and 15 comes from a single cycle of g_v .

$$\text{The } |Stab_G([1,2,3], [7,8,9], [13,14,15])| = 27.$$

By Orbit-Stabilizer Theorem,

$$\begin{aligned} |Orb_G([1,2,3], [7,8,9], [13,14,15])| &= |G : Stab_G([1,2,3], [7,8,9], [13,14,15])| \\ &= \frac{|G|}{|Stab_G([1,2,3], [7,8,9], [13,14,15])|} \\ &= \frac{46\ 656\ 000}{27} = 1728000 = |P^{[3]} \times S^{[3]} \times V^{[3]}| \end{aligned}$$

Therefore, $A_6 \times A_6 \times A_6$ acts transitively on $P^{[3]} \times S^{[3]} \times V^{[3]}$.

Lemma 2.3: The action of $A_7 \times A_7 \times A_7$ on $P^{[3]} \times S^{[3]} \times V^{[3]}$ is transitive.

Proof: Let $G = A_7 \times A_7 \times A_7$ act on $P^{[3]} \times S^{[3]} \times V^{[3]}$ where;

gap> Arrangements([1,2,3,4,5,6,7],3); $P^{[3]} = \{ [1, 2, 3], [1, 2, 4], [1, 2, 5], [1, 2, 6], [1, 2, 7], [1, 3, 2], [1, 3, 4], [1, 3, 5], [1, 3, 6], [1, 3, 7], [1, 4, 2], [1, 4, 3], [1, 4, 5], [1, 4, 6], [1, 4, 7], [1, 5, 2], [1, 5, 3], [1, 5, 4], [1, 5, 6], [1, 5, 7], [1, 6, 2], [1, 6, 3], [1, 6, 4], [1, 6, 5], [1, 6, 7], [1, 7, 2], [1, 7, 3], [1, 7, 4], [1, 7, 5], [1, 7, 6], [2, 1, 3], [2, 1, 4], [2, 1, 5], [2, 1, 6], [2, 1, 7], [2, 3, 1], [2, 3, 4], [2, 3, 5], [2, 3, 6], [2, 3, 7], [2, 4, 1], [2, 4, 3], [2, 4, 5], [2, 4, 6], [2, 4, 7], [2, 5, 1], [2, 5, 3], [2, 5, 4], [2, 5, 6], [2, 5, 7], [2, 6, 1], [2, 6, 3], [2, 6, 4], [2, 6, 5], [2, 6, 7], [2, 7, 1], [2, 7, 3], [2, 7, 4], [2, 7, 5], [2, 7, 6], [3, 1, 2], [3, 1, 4], [3, 1, 5], [3, 1, 6], [3, 1, 7], [3, 2, 1], [3, 2, 4], [3, 2, 5], [3, 2, 6], [3, 2, 7], [3, 4, 1], [3, 4, 2], [3, 4, 5], [3, 4, 6], [3, 4, 7], [3, 5, 1], [3, 5, 2], [3, 5, 4], [3, 5, 6], [3, 5, 7], [3, 6, 1], [3, 6, 2], [3, 6, 4], [3, 6, 5], [3, 6, 7], [3, 7, 1], [3, 7, 2], [3, 7, 4], [3, 7, 5], [3, 7, 6], [4, 1, 2], [4, 1, 3], [4, 1, 5], [4, 1, 6], [4, 1, 7], [4, 2, 1], [4, 2, 3], [4, 2, 5], [4, 2, 6], [4, 2, 7], [4, 3, 1], [4, 3, 2], [4, 3, 4], [4, 3, 5], [4, 3, 6], [4, 3, 7], [4, 4, 1], [4, 4, 2], [4, 4, 3], [4, 4, 4], [4, 4, 5], [4, 4, 6], [4, 4, 7], [4, 5, 1], [4, 5, 2], [4, 5, 3], [4, 5, 4], [4, 5, 5], [4, 5, 6], [4, 5, 7], [4, 6, 1], [4, 6, 2], [4, 6, 3], [4, 6, 4], [4, 6, 5], [4, 6, 6], [4, 6, 7], [4, 7, 1], [4, 7, 2], [4, 7, 3], [4, 7, 4], [4, 7, 5], [4, 7, 6], [4, 7, 7], [5, 1, 2], [5, 1, 3], [5, 1, 4], [5, 1, 5], [5, 1, 6], [5, 1, 7], [5, 2, 1], [5, 2, 2], [5, 2, 3], [5, 2, 4], [5, 2, 5], [5, 2, 6], [5, 2, 7], [5, 3, 1], [5, 3, 2], [5, 3, 3], [5, 3, 4], [5, 3, 5], [5, 3, 6], [5, 3, 7], [5, 4, 1], [5, 4, 2], [5, 4, 3], [5, 4, 4], [5, 4, 5], [5, 4, 6], [5, 4, 7], [5, 5, 1], [5, 5, 2], [5, 5, 3], [5, 5, 4], [5, 5, 5], [5, 5, 6], [5, 5, 7], [5, 6, 1], [5, 6, 2], [5, 6, 3], [5, 6, 4], [5, 6, 5], [5, 6, 6], [5, 6, 7], [5, 7, 1], [5, 7, 2], [5, 7, 3], [5, 7, 4], [5, 7, 5], [5, 7, 6], [5, 7, 7], [6, 1, 2], [6, 1, 3], [6, 1, 4], [6, 1, 5], [6, 1, 6], [6, 1, 7], [6, 2, 1], [6, 2, 2], [6, 2, 3], [6, 2, 4], [6, 2, 5], [6, 2, 6], [6, 2, 7], [6, 3, 1], [6, 3, 2], [6, 3, 3], [6, 3, 4], [6, 3, 5], [6, 3, 6], [6, 3, 7], [6, 4, 1], [6, 4, 2], [6, 4, 3], [6, 4, 4], [6, 4, 5], [6, 4, 6], [6, 4, 7], [6, 5, 1], [6, 5, 2], [6, 5, 3], [6, 5, 4], [6, 5, 5], [6, 5, 6], [6, 5, 7], [6, 6, 1], [6, 6, 2], [6, 6, 3], [6, 6, 4], [6, 6, 5], [6, 6, 6], [6, 6, 7], [6, 7, 1], [6, 7, 2], [6, 7, 3], [6, 7, 4], [6, 7, 5], [6, 7, 6], [6, 7, 7], [7, 1, 2], [7, 1, 3], [7, 1, 4], [7, 1, 5], [7, 1, 6], [7, 1, 7], [7, 2, 1], [7, 2, 2], [7, 2, 3], [7, 2, 4], [7, 2, 5], [7, 2, 6], [7, 2, 7], [7, 3, 1], [7, 3, 2], [7, 3, 3], [7, 3, 4], [7, 3, 5], [7, 3, 6], [7, 3, 7], [7, 4, 1], [7, 4, 2], [7, 4, 3], [7, 4, 4], [7, 4, 5], [7, 4, 6], [7, 4, 7], [7, 5, 1], [7, 5, 2], [7, 5, 3], [7, 5, 4], [7, 5, 5], [7, 5, 6], [7, 5, 7], [7, 6, 1], [7, 6, 2], [7, 6, 3], [7, 6, 4], [7, 6, 5], [7, 6, 6], [7, 6, 7], [7, 7, 1], [7, 7, 2], [7, 7, 3], [7, 7, 4], [7, 7, 5], [7, 7, 6], [7, 7, 7] \}$

5], [4, 1, 6], [4, 1, 7], [4, 2, 1], [4, 2, 3], [4, 2, 5], [4, 2, 6], [4, 2, 7], [4, 3, 1], [4, 3, 2], [4, 3, 5], [4, 3, 6], [4, 3, 7], [4, 5, 1], [4, 5, 2], [4, 5, 3], [4, 5, 6], [4, 5, 7], [4, 6, 1], [4, 6, 2], [4, 6, 3], [4, 6, 5], [4, 6, 7], [4, 7, 1], [4, 7, 2], [4, 7, 3], [4, 7, 5], [4, 7, 6], [5, 1, 2], [5, 1, 3], [5, 1, 4], [5, 1, 6], [5, 1, 7], [5, 2, 1], [5, 2, 3], [5, 2, 4], [5, 2, 6], [5, 2, 7], [5, 3, 1], [5, 3, 2], [5, 3, 4], [5, 3, 6], [5, 3, 7], [5, 4, 1], [5, 4, 2], [5, 4, 3], [5, 4, 6], [5, 4, 7], [5, 6, 1], [5, 6, 2], [5, 6, 3], [5, 6, 4], [5, 6, 7], [5, 7, 1], [5, 7, 2], [5, 7, 3], [5, 7, 4], [5, 7, 6], [6, 1, 2], [6, 1, 3], [6, 1, 4], [6, 1, 5], [6, 1, 7], [6, 2, 1], [6, 2, 3], [6, 2, 4], [6, 2, 5], [6, 2, 7], [6, 3, 1], [6, 3, 2], [6, 3, 4], [6, 3, 5], [6, 3, 7], [6, 4, 1], [6, 4, 2], [6, 4, 3], [6, 4, 5], [6, 4, 7], [6, 5, 1], [6, 5, 2], [6, 5, 3], [6, 5, 4], [6, 5, 7], [6, 7, 1], [6, 7, 2], [6, 7, 3], [6, 7, 4], [6, 7, 5], [7, 1, 2], [7, 1, 3], [7, 1, 4], [7, 1, 5], [7, 1, 6], [7, 2, 1], [7, 2, 3], [7, 2, 4], [7, 2, 5], [7, 2, 6], [7, 3, 1], [7, 3, 2], [7, 3, 4], [7, 3, 5], [7, 3, 6], [7, 4, 1], [7, 4, 2], [7, 4, 3], [7, 4, 5], [7, 4, 6], [7, 5, 1], [7, 5, 2], [7, 5, 3], [7, 5, 4], [7, 5, 6], [7, 6, 1], [7, 6, 2], [7, 6, 3], [7, 6, 4], [7, 6, 5];

gap> Arrangements([8,9,10,11,12,13,14],3); $S^{[3]} = \{ [8, 9, 10], [8, 9, 11], [8, 9, 12], [8, 9, 13], [8, 9, 14], [8, 10, 9], [8, 10, 11], [8, 10, 12], [8, 10, 13], [8, 10, 14], [8, 11, 9], [8, 11, 10], [8, 11, 12], [8, 11, 13], [8, 11, 14], [8, 12, 9], [8, 12, 10], [8, 12, 11], [8, 12, 13], [8, 12, 14], [8, 13, 9], [8, 13, 10], [8, 13, 11], [8, 13, 12], [8, 13, 14], [8, 14, 9], [8, 14, 10], [8, 14, 11], [8, 14, 12], [8, 14, 13], [9, 8, 10], [9, 8, 11], [9, 8, 12], [9, 8, 13], [9, 8, 14], [9, 10, 8], [9, 10, 11], [9, 10, 12], [9, 10, 13], [9, 10, 14], [9, 11, 8], [9, 11, 10], [9, 11, 12], [9, 11, 13], [9, 11, 14], [9, 12, 8], [9, 12, 10], [9, 12, 11], [9, 12, 13], [9, 12, 14], [9, 13, 8], [9, 13, 10], [9, 13, 11], [9, 13, 12], [9, 13, 14], [9, 14, 8], [9, 14, 10], [9, 14, 11], [9, 14, 12], [9, 14, 13], [10, 8, 9], [10, 8, 11], [10, 8, 12], [10, 8, 13], [10, 8, 14], [10, 9, 8], [10, 9, 11], [10, 9, 12], [10, 9, 13], [10, 9, 14], [10, 11, 8], [10, 11, 9], [10, 11, 12], [10, 11, 13], [10, 11, 14], [10, 12, 8], [10, 12, 9], [10, 12, 11], [10, 12, 13], [10, 12, 14], [10, 13, 8], [10, 13, 9], [10, 13, 11], [10, 13, 12], [10, 13, 14], [10, 14, 8], [10, 14, 9], [10, 14, 11], [10, 14, 12], [10, 14, 13], [11, 8, 9], [11, 8, 10], [11, 8, 12], [11, 8, 13], [11, 8, 14], [11, 9, 8], [11, 9, 10], [11, 9, 12], [11, 9, 13], [11, 9, 14], [11, 10, 8], [11, 10, 9], [11, 10, 12], [11, 10, 13], [11, 10, 14], [11, 12, 8], [11, 12, 9], [11, 12, 10], [11, 12, 13], [11, 12, 14], [11, 13, 8], [11, 13, 9], [11, 13, 10], [11, 13, 12], [11, 13, 14], [11, 14, 8], [11, 14, 9], [11, 14, 10], [11, 14, 12], [11, 14, 13], [12, 8, 9], [12, 8, 10], [12, 8, 11], [12, 8, 13], [12, 8, 14], [12, 9, 8], [12, 9, 10], [12, 9, 11], [12, 9, 13], [12, 9, 14], [12, 10, 8], [12, 10, 9], [12, 10, 11], [12, 10, 13], [12, 10, 14], [12, 11, 8], [12, 11, 9], [12, 11, 10], [12, 11, 13], [12, 11, 14], [12, 13, 8], [12, 13, 9], [12, 13, 10], [12, 13, 11], [12, 13, 14], [12, 14, 8], [12, 14, 9], [12, 14, 10], [12, 14, 11], [12, 14, 13], [13, 8, 9], [13, 8, 10], [13, 8, 11], [13, 8, 12], [13, 8, 14], [13, 9, 8], [13, 9, 10], [13, 9, 11], [13, 9, 12], [13, 9, 14], [13, 10, 8], [13, 10, 9], [13, 10, 11], [13, 10, 12], [13, 10, 14], [13, 11, 8], [13, 11, 9], [13, 11, 10], [13, 11, 12], [13, 11, 14], [13, 12, 8], [13, 12, 9], [13, 12, 10], [13, 12, 11], [13, 12, 14], [13, 14, 8], [13, 14, 9], [13, 14, 10], [13, 14, 11], [13, 14, 12], [14, 8, 9], [14, 8, 10], [14, 8, 11], [14, 8, 12], [14, 8, 13], [14, 9, 8], [14, 9, 10], [14, 9, 11], [14, 9, 12], [14, 9, 13], [14, 10, 8], [14, 10, 9], [14, 10, 11], [14, 10, 12], [14, 10, 13], [14, 11, 8], [14, 11, 9], [14, 11, 10], [14, 11, 12], [14, 11, 13], [14, 12, 8], [14, 12, 9], [14, 12, 10], [14, 12, 11], [14, 12, 13], [14, 13, 8], [14, 13, 9], [14, 13, 10], [14, 13, 11], [14, 13, 12];$

and

gap> Arrangements([15,16,17,18,19,20,21],3); $V^{[3]} = \{ [15, 16, 17], [15, 16, 18], [15, 16, 19], [15, 16, 20], [15, 16, 21], [15, 17, 16], [15, 17, 18], [15, 17, 19], [15, 17, 20], [15, 17, 21], [15, 18, 16], [15, 18, 17], [15, 18, 19], [15, 18, 20], [15, 18, 21], [15, 19, 16], [15, 19, 17], [15, 19, 18], [15, 19, 20], [15, 19, 21], [15, 20, 16], [15, 20, 17], [15, 20, 18], [15, 20, 19], [15, 20, 21], [15, 21, 16], [15, 21, 17], [15, 21, 18], [15, 21, 19], [15, 21, 20], [16, 15, 17], [16, 15, 18], [16, 15, 19], [16, 15, 20], [16, 15, 21], [16, 17, 15], [16, 17, 18], [16, 17, 19], [16, 17, 20], [16, 17, 21], [16, 18, 15], [16, 18, 17], [16, 18, 19], [16, 18, 20], [16, 18, 21], [16, 19, 15], [16, 19, 17], [16, 19, 18], [16, 19, 20], [16, 19, 21], [16, 20, 15], [16, 20, 17], [16, 20, 18], [16, 20, 19], [16, 20, 21], [16, 21, 15], [16, 21, 17], [16, 21, 18], [16, 21, 19], [16, 21, 20], [17, 15, 16], [17, 15, 18], [17, 15, 19], [17, 15, 20], [17, 15, 21], [17, 16, 15], [17, 16, 18], [17, 16, 19], [17, 16, 20], [17, 16, 21], [17, 18, 15], [17, 18, 16], [17, 18, 19], [17, 18, 20], [17, 18, 21], [17, 19, 15], [17, 19, 16], [17, 19, 18], [17, 19, 20], [17, 19, 21], [17, 20, 15], [17, 20, 16], [17, 20, 18], [17, 20, 19], [17, 20, 21], [17, 21, 15], [17, 21, 16], [17, 21, 18], [17, 21, 19], [17, 21, 20], [18, 15, 16], [18, 15, 17], [18, 15, 19], [18, 15, 20], [18, 15, 21], [18, 16, 15], [18, 16, 17], [18, 16, 19], [18, 16, 20], [18, 16, 21], [18, 17, 15], [18, 17, 16], [18, 17, 19], [18, 17, 20], [18, 17, 21], [18, 19, 15], [18, 19, 16], [18, 19, 17], [18, 19, 20], [18, 19, 21], [18, 20, 15], [18, 20, 16], [18, 20, 17], [18, 20, 19], [18, 20, 21], [18, 21, 15], [18, 21, 16], [18, 21, 17], [18, 21, 19], [18, 21, 20], [19, 15, 16], [19, 15, 17], [19, 15, 18], [19, 15, 20], [19, 15, 21], [19, 16, 15], [19, 16, 17], [19, 16, 18], [19, 16, 20], [19, 16, 21], [19, 17, 15], [19, 17, 16], [19, 17, 18], [19, 17, 20], [19, 17, 21], [19, 18, 15], [19, 18, 16], [19, 18, 17], [19, 18, 20], [19, 18, 21], [19, 20, 15], [19, 20, 16], [19, 20, 17], [19, 20, 18], [19, 20, 21], [19, 21, 15], [19, 21, 16], [19, 21, 17], [19, 21, 18], [19, 21, 19], [19, 21, 20], [19, 21, 21];$

19, 21, 18], [19, 21, 20], [20, 15, 16], [20, 15, 17], [20, 15, 18], [20, 15, 19], [20, 15, 21], [20, 16, 15], [20, 16, 17], [20, 16, 18], [20, 16, 19], [20, 16, 21], [20, 17, 15], [20, 17, 16], [20, 17, 18], [20, 17, 19], [20, 17, 21], [20, 18, 15], [20, 18, 16], [20, 18, 17], [20, 18, 19], [20, 18, 21], [20, 19, 15], [20, 19, 16], [20, 19, 17], [20, 19, 18], [20, 19, 21], [20, 21, 15], [20, 21, 16], [20, 21, 17], [20, 21, 18], [20, 21, 19], [21, 15, 16], [21, 15, 17], [21, 15, 18], [21, 15, 19], [21, 15, 20], [21, 16, 15], [21, 16, 17], [21, 16, 18], [21, 16, 19], [21, 16, 20], [21, 17, 15], [21, 17, 16], [21, 17, 18], [21, 17, 19], [21, 17, 20], [21, 18, 15], [21, 18, 16], [21, 18, 17], [21, 18, 19], [21, 18, 20], [21, 19, 15], [21, 19, 16], [21, 19, 17], [21, 19, 18], [21, 19, 20], [21, 20, 15], [21, 20, 16], [21, 20, 17], [21, 20, 18], [21, 20, 19]}.

The cartesian product of $P^{[3]} \times S^{[3]} \times V^{[3]}$ is generated using the GAP software with, $|P^{[3]} \times S^{[3]} \times V^{[3]}| = 9\,261\,000$. G is generated by

$\langle \{(1234567), (123)\}, \{(8\,9\,10\,11\,12\,13\,14), (8\,9\,10)\}, \{(15\,16\,17\,18\,19\,20\,21), (15\,16\,17)\} \rangle$ using the GAP software. $([1,2,3], [8,9,10], [15,16,17])$ is fixed by an element $(g_p, g_s, g_v) \in G$ if and only if 1,2 and 3 comes from a single cycle of g_p ; 8,9 and 10 comes from a single cycle of g_s and 15,16 and 17 comes from a single cycle of g_v .

$$\text{The } |Stab_G([1,2,3], [8,9,10], [15,16,17])| = 1728.$$

By Orbit-Stabilizer Theorem,

$$\begin{aligned} |Orb_G([1,2,3], [8,9,10], [15,16,17])| &= |G: Stab_G([1,2,3], [8,9,10], [15,16,17])| \\ &= \frac{|G|}{|Stab_G([1,2,3], [8,9,10], [15,16,17])|} \\ &= \frac{16\,003\,008\,000}{1728} = 9261000 = |P^{[3]} \times S^{[3]} \times V^{[3]}| \end{aligned}$$

Therefore, $A_7 \times A_7 \times A_7$ acts transitively on $P^{[3]} \times S^{[3]} \times V^{[3]}$.

Theorem 2.4: The action of $A_n \times A_n \times A_n$ on $P^{[3]} \times S^{[3]} \times V^{[3]}$ is transitive if and only if $n \geq 5$.

Proof: Let $G = G_p \times G_s \times G_v = A_n \times A_n \times A_n$ act on $P^{[3]} \times S^{[3]} \times V^{[3]}$. It suffices to verify that $|P^{[3]} \times S^{[3]} \times V^{[3]}|$ is equal to $|Orb_G([1,2,3], [n+1, n+2, n+3], [2n+1, 2n+2, 2n+3])|$.

$$\text{Let } |R| = |Stab_G([1,2,3], [n+1, n+2, n+3], [2n+1, 2n+2, 2n+3])|.$$

So, $(g_p, g_s, g_v) \in G = A_n \times A_n \times A_n$ fixes $([1,2,3], [n+1, n+2, n+3], [2n+1, 2n+2, 2n+3]) \in P^{[3]} \times S^{[3]} \times V^{[3]}$ if and only if 1,2 and 3 comes from 1-cycle of g_p ; $n+1, n+2$ and $n+3$ comes from 1-cycle of g_s and $2n+1, 2n+2$ and $2n+3$ comes from 1-cycle of g_v .

The $Stab_G([1,2,3], [n+1, n+2, n+3], [2n+1, 2n+2, 2n+3])$ is isomorphic to: $A_{n-3} \times A_{n-3} \times A_{n-3}$.

Therefore, $|R| = |Stab_G([1,2,3], [n+1, n+2, n+3], [2n+1, 2n+2, 2n+3])| = |Stab_{G_p}([1,2,3]) \times Stab_{G_s}([n+1, n+2, n+3]) \times Stab_{G_v}([2n+1, 2n+2, 2n+3])|$

$$|R| = \frac{(n-3)! \times (n-3)! \times (n-3)!}{2 \times 2 \times 2} = \left(\frac{(n-3)!}{2}\right)^3$$

Applying the Orbit-Stabilizer Theorem we get;

$$\begin{aligned}
 &|Orb_G([1,2,3], [n + 1, n + 2, n + 3], [2n + 1, 2n + 2, 2n + 3])| \\
 &= |G: Stab_G([1,2,3], [n + 1, n + 2, n + 3], [2n + 1, 2n + 2, 2n + 3])| \\
 &|G| = \frac{n! \times n! \times n!}{2 \times 2 \times 2} = \left(\frac{n!}{2}\right)^3 \\
 &\frac{|G|}{|R|} = \frac{\left(\frac{n!}{2}\right)^3}{\left(\frac{(n-3)!}{2}\right)^3} = \left(\frac{n!}{(n-3)!}\right)^3.
 \end{aligned}$$

Therefore;

$$\frac{|G|}{|R|} = \left(\frac{n!}{(n-3)!}\right)^3 = |P^{[3]} \times S^{[3]} \times V^{[3]}|$$

Hence, $A_n \times A_n \times A_n$ acts transitively on $P^{[3]} \times S^{[3]} \times V^{[3]}$ if $n \geq 5$.

Corollary 2.5: For $n < 5$, the

$$|Stab_G([1,2,3], [n + 1, n + 2, n + 3], [2n + 1, 2n + 2, 2n + 3])| = |A_{n-3} \times A_{n-3} \times A_{n-3}| < 1.$$

4. Conclusion

The cartesian product of the alternating group $A_n (n \geq 5)$ acting on a cartesian product of ordered sets of triples has been determined to be transitive using the Orbit-Stabilizer Theorem by showing that the length of the orbit (p, s, v) in $A_n \times A_n \times A_n$ ($n \geq 5$) acting on $P^{[3]} \times S^{[3]} \times V^{[3]}$ is equivalent to the cardinality of $P^{[3]} \times S^{[3]} \times V^{[3]}$ to imply transitivity.

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Competing Interests

Authors have declared that no competing interests exist.

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