

CHUKA

UNIVERSITY



## UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF SCIENCE IN  
PHYSICS

PHYS 841: ELECTRODYNAMICS

STREAMS: MSc.

TIME: 3 HOURS

DAY/DATE: MONDAY 14/04/2025

2.30 P.M. – 5.30 A.M.

**INSTRUCTIONS**

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m.}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg.}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

**Answer question ANY FOUR questions****QUESTION ONE (15 MARKS)**

- (a) Using Maxwell's equations show that the time varying fields, there will be a surface current density  $K$  due to the tangential magnetic field such that  $n \times H = K$  (3 marks)
- (b) Derive Laplace's equation from Gauss's Law and describe how Laplace's equation governs the behavior of conductors in electrostatic equilibrium. (4 marks)
- (c) A thin, non-conducting rod of length  $L=2$  m carries a linear charge density  $\lambda=10^{-6}$  C/m. Derive the expression for the electric field at a point P located on the perpendicular bisector of the rod at a distance  $d=0.5$  m from its center. (3 marks)
- (d) A parallel plate capacitor with plate area  $A=0.1$  m<sup>2</sup> and plate separation  $d=1$  mm is filled with a dielectric of permittivity  $\epsilon_r=3$ . If the plates are maintained at a potential difference of  $V=500$  V, determine: (i) The capacitance of the capacitor, (ii) The energy stored in the capacitor and (iii) The energy density in the dielectric medium. (3 marks)
- (e) Describe one applications of Reflection and one applications Refraction of EM waves (2 marks)

**QUESTION TWO (15 MARKS)**

- a. State and prove Green's Theorem in two dimensions. (3 marks)
- b. Derive the Green's function for Laplace's equation in a half-space problem, where a conducting plane at  $z=0$  enforces  $V(x,y,0)=0$ . (3 marks)
- c. Explain the concept of multipole expansion and derive the general expression for the electric potential due to a localized charge distribution. (3 marks)
- d. A conducting medium with conductivity  $\sigma$  and permittivity  $\epsilon$  modifies Maxwell's equations.
  - (i) Derive the modified wave equation for the electric field in a conducting medium.
  - (ii) Define the skin depth  $\delta$  and derive its expression in terms of  $\sigma$  and  $\omega$ .
  - (iii) Calculate the skin depth for a metal with  $\sigma=5.8 \times 10^7$  S/m at a frequency of 1 GHz. (6 marks)

**QUESTION THREE (15 MARKS)**

- (a) State Maxwell's equations in differential forms and explain the physical significance of each equation (4 marks)
- (b) Derive the wave equation for the electric field  $E$  in free space. And determine the speed of light in vacuum using the fundamental constants  $\mu_0$  and  $\epsilon_0$  (5 marks)
- (c) An electromagnetic wave, travelling in a medium with relative permittivity  $\epsilon$ , relative permeability  $\mu$  and refractive index  $n$  and whose electric component is parallel to the plane of incidence, encounters another medium with relative permittivity  $\epsilon'$ , relative permeability  $\mu'$ . Show that the
  - (i) fraction of refracted electric component is given by

$$\frac{E'_o}{E_o} = \frac{2nn' \cos i}{\frac{\mu}{\mu'} n'^2 \cos i + n \sqrt{n'^2 - n^2 \sin^2 i}}$$

6 marks

**QUESTION FOUR (15 MARKS)**

- (a) Explain the concept of gauge freedom in electrodynamics and its extension to quantum electrodynamics (QED). (3 marks)
- (b) Define the Lorenz Gauge and explain how it differs from the Coulomb Gauge. (2 marks)

- (c) A plane electromagnetic wave is incident normally from air ( $n_1=1$ ) onto a glass surface ( $n_2=1.5$ ). The incident wave has an electric field amplitude of  $E_0=100$  V/m. (i) Compute the reflection coefficient  $R$  and transmission coefficient  $T$  using Fresnel equations (ii) Determine the amplitudes of the reflected and transmitted electric fields. (4 marks)
- (d) A plasma has an electron density of  $N_e=10^{12}$  electrons/cm<sup>3</sup>. Compute the plasma frequency  $\omega_p$  in Hz and determine whether a wave with frequency  $f=10$  GHz can propagate through this plasma. (4 marks)
- (e) Differentiate between wave guide and cavity (2 marks)

**QUESTION FIVE (15 MARKS)**

- (a) Derive the wave equation for the electric and magnetic fields in a rectangular waveguide starting from Maxwell's equations and show that the solutions take the form of transverse electric (TE) and transverse magnetic (TM) modes. (4 marks)
- (b) A rectangular waveguide has dimensions  $a=4$  cm and  $b=2$  cm. Calculate the cutoff frequency for the lowest-order TE mode ( $TE_{10}$ ) and determine whether a signal with frequency  $f=6$  GHz can propagate in this waveguide. (3 marks)
- (c) rectangular cavity resonator with dimensions  $a=5$  cm,  $b=3$  cm,  $d=2$  cm, filled with a material of conductivity  $\sigma=5.8 \times 10^7$  S/m, (i) Compute the resonant frequency for the  $TE_{101}$  mode, (ii) Determine the skin depth and the power loss at this frequency and (iii) Calculate the quality factor  $Q$  and discuss its significance in selecting materials for cavity (6 marks)
- (d) Show that the radiated power is given by the Larmor formula. (2 marks)

$$P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3}$$


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