CHUKA **UNIVERSITY**

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN APPLIED COMPUTER SCIENCE

ACSC 271: MATHEMATICAL METHODS FOR COMPUTER SCIENTISTS

STREAMS: BSC **TIME: 2 HOURS**

DAY/DATE: MONDAY 16/12/2024 **11.30 A.M – 1.30 P.M.**

INSTRUCTIONS

- **Answer Question ONE and any other TWO Questions.**
- **Adhere to the instructions on the answer booklet.**

QUESTION ONE (30 MARKS)

a. Given the function defined as $f(x)=\begin{cases}2x-1 \ if \ x>0\\ 2x^2-1 \ if \ x\le 0\end{cases}$ Sketch the graph of the function and obtain $f(0)$ and $f(2)$ (3 marks)

b. Obtain the derivative of the function $y=f(x)=2x^2+4$, from first principles. (3 marks)

c. State the order, linearity and degree of the following differential equation. (3 marks)

$$x+\left(\frac{dy}{dx}\right)^2=\frac{d^2y}{dx^2}+y^2$$

d. Show that the vectors $\vec{a}=3i+2j+k,\ \vec{b}=-2i+3j+5k$, are not orthogonal hence find the angle between the vectors (3 marks)

e. Evaluate the following limits:

i. $\lim_{x\to\infty} \dfrac{x^3+6x^2-3}{5x^3+1}$ (3 marks)

ii. $\lim_{x\to 2} \dfrac{x^2+3x-10}{x-2}$ (4 marks)

f. Obtain the gradient of the curve $y=\dfrac{1}{x}$, at the point x = 1 (4 marks)

g. Given that $y=\sin^{-1}(2x+3)$, find $\dfrac{dy}{dx}$ (4 marks)

h. Verify whether $y=\sin 2x+5x+10$ is a solution of $\dfrac{dy}{dx}-2\cos x=5$ (3 marks)

QUESTION TWO (20 MARKS)

a. Find the domain of the function below $f(x)=\sqrt{3x^2-3x-18}$ (4 marks)

b. Verify whether the differential equation $2\left(x^2+y^2\right)dx+4xydy=0$ is exact, Hence find the particular integral given that $y(1)=1$ (5 marks)

c. Given the differential equation $y''-6y'+9y=0$, state the order and linearity Hence solve by the D Operator (5 marks)

d. Solve the simultaneous equations below both by the Cramer's rule and the row reduction Method.

$3x+4y+z=10$
$2x-3y+5z=-9$
$x+2y-z=6$

(6 marks)

QUESTION THREE (20 MARKS)

a) Evaluate the definite integral $\int_0^1 9x^2\sqrt{3x^3+5}\,dx$ (5 marks)

b) Given the vertices of a triangle as $A(2,3,1), B(4,0,6), C(3,2,4)$, Evaluate the area of thetriangle

(5 marks)

c) Prove the divergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ by the ratio test (5 marks)

d) Find the equation of the tangent to the curve $y^2 = e^{2x+1}$ at the point x = 0. (5 marks)

QUESTION FOUR (20 MARKS)

a. Given the integral $y = \int_1^2 \left(2x^2 + 5\right) dx$, Use the trapezoidal rule with 3 strips (n = 3) to approximate the integral and obtain the actual error (6 marks)

b. Verify whether the differential equation below is exact hence Solve it.
$\left(4x^3y^3 + 3x^2\right)dx + \left(3x^4y^2 + 6y^2\right)dy = 0$
(4 marks)

c. Find the equations of lines tangent to the curve $y^2 - 6x^2 + 4y + 6 = 0$. at the point y = 0
(5 marks)

d. Find the area of the parallelogram formed by the vectors $A = 3i + j - 2k,\ B = i - 3j - 2k$
(5 marks)

QUESTION FIVE (20 MARKS)

a) Prove the divergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^n}$ by the root test (3 marks)

b) Solve the following ordinary differential equations by the D Operator
$y'' - 4y' + 3y = 0$
(5 marks)

c) Find the determinant, the characteristic equation and the Eigen values of the matrix

$$\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

(6 marks)

d) Discuss the consistency of the following system of equations using row reduction method hence solve it if found consistent. (6 marks)

$$x + y + z = 3$$
$$x + 5y + 2z = 8$$
$$3x - y + 3z = 5$$
