CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE

MATH 423: NUMERICAL ANALYSIS II

STREAMS: AB1, AB5, EB16, EB2 **TIME: 2 HOURS**

DAY/DATE: WEDNESDAY 20/12/2023 **2.30 P.M. – 4.30 P.M.**

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a. Find the eigenvalues and eigenvectors of the matrix A given that $A=\begin{pmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{pmatrix}$ (5 Marks)

b. Use the condition number to determine whether or not the matrix is well or ill conditioned

$$A=\begin{pmatrix} 0.1 & 2 \\ 1 & 2 \end{pmatrix}$$ (4 Marks)

c. Construct the simple linear regression equation of y on x given that

$$n=7, \sum_{i=1}^{n} x_i = 113, \sum_{i=1}^{n} x_i{}^{2}{}_{i} = 1983, \sum_{i=1}^{n} y_i = 183 \,,\,, \sum_{i=1}^{n} x_i y_i = 3186$$ (6 Marks)

d. Use lagrange's inverse interpolation formula to find x, given that y=15 from the data in the table below (5 Marks)

x	5	6	9	11
y	12	13	14	16

e. For the matrix $A=\begin{pmatrix} -6 & 3 \\ 4 & 5 \end{pmatrix}$, find the EigenVector corresponding to the EigenValue $\lambda=6$ (5 Marks)

f. Express x^5 in terms of Chebyshev polynomials (T_n) (5 Marks)

QUESTION TWO (20 MARKS)

a. Use the Chebyshev recurrence formula $T_{(n+1)}(x) = 2\,x\,T_n\,(x) - T_{(n-1)}(x)$ to generate $T_4(x)$ (5 Marks)

b. Use the Jacobi Iterative method with $x^0 = (0,0,0)^T$ to solve the system of equations to 0ne significant figure after 4 iterations (8 Marks)

$$8x - 3y + 2z = 20$$
$$4x - 11y - z = 33$$
$$6x + 3y + 12z = 35$$

c. Fit a second order polynomial to the data in the table below using the least square method (7 Marks)

x	0	1	2	3	4	5
y	2.1	7.7	13.6	27.2	40.9	61.1

QUESTION THREE (20 MARKS)

a) Use Hermites interpolation based on divided differences to approximate $\sqrt{45}$ to 5decimal places

(9 Marks)

b) Using Crowts LU decomposition method to solve the system (11 Marks)

$$5x_1 - 2x_2 + x_3 = 4$$
$$7x_1 + x_2 - 5z = 8$$
$$3x_1 + 7x_2 + 4z = 10$$

QUESTION FOUR (2 MARKS)

a. The LU decomposition of a matrix A is L= $\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 1 & 1 \end{pmatrix}$ and U= $\begin{pmatrix} 1 & 4 & 3 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{pmatrix}$

i. Find A (2 Marks)

ii. Use LU decomposition to solve the system $A\vec{x} = \vec{B}$ where $\vec{B} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ (5 Marks)

b. Use the power method with $x^0 = (1,1,1)^T$ to find the largest Eigenvalue and the corresponding Eigenvector for the matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ (9 Marks)

c. Consider the matrix A given as $A = \begin{pmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 13 & 1 \end{pmatrix}$

i. Determine the diagonal dominance of the matrix A (3 Marks)

ii. State the importance of diagonal dominance in matrices (1 Marks)

QUESTION FIVE (20 MARKS)

a. Use the Gauss Seidel method with $x^0 = (1,1,0)^T$ to solve the system of Linear Equations (10 Marks)

$$x_1 + x_2 + 4x_3 = 9$$
$$8x_1 - 3x_2 + 2x_3 = 20$$
$$4x_1 + 11x_2 - x_3 = 33$$

a. A car travelling along a straight road is clocked at a number of points. The data from the observations is given in the table below where the time is in seconds, the distance in feet and the speed which is $\frac{ds}{dt}$ is in feet/sec.

Time (t)	0	3	5
Distance(s)	0	225	383
Speed= $\frac{ds}{dt}$	75	77	80

Use a Hermite polynomial $H_{2n+1}(t)$ to predict the position to predict the position of the car and its speed when $t = 4\sec$ (10 Marks)

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