

CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS: 2023

EXAMINATION FOR THE AWARD OF DEGREE OF DOCTOR OF PHILOSOPHY IN
SCIENCE :APPLIED MATHEMATICS.

MATH 831: METHODS OF APPLIED MATHS II

TIME: 2 HOURS.

INSTRUCTIONS

Answer any THREE Questions.

QUESTION ONE

a. Prove that $(n+1)P_{n+1} = (2n+1)xP_n - nP_{n-1}$, where $P_n(x)$ is the Legendres Polynomial 6mks

b. Find the shape of the curve of the given perimeter enclosing maximum area 8mks

c. Show that 7mks

$$\sqrt{\left(\frac{1}{2}\pi x\right)} J_{\frac{3}{2}}(x) = \frac{\sin x}{x} - \cos x$$

QUESTION TWO

a. Express $J_4(x)$ in terms of $J_1(x)$ and $J_0(x)$ given that 5mks

b. Prove that 6mks

$$J_{5/2}(x) = \sqrt{\frac{2}{\pi x}} \left[\left(\frac{3-x^2}{x^2} \right) \sin x - \frac{3 \cos x}{x} \right]$$

c. Find the curve connecting the points (x_1, y_1) and (x_2, y_2) which when rotated about the x axis gives a minimum surface 9mks

QUESTION THREE

a. Express $f(x) = 4x^3 + 6x^2 + 7x + 2$ in terms of Legendre Polynomials 5mks

b. Solve the integral equation below 5mks

$$y(x) = x + \lambda \int_0^1 (xz + z^2)y(z) dz.$$

- c. Find the curve connecting the points (x_1, y_1) and (x_2, y_2) which when rotated about the x axis gives a minimum surface 9mks

QUESTION FOUR

- a. Prove that $J_n(x)$ is the coefficient of z^n in the expansion of 6mks

$$e^{\frac{x}{2}\left(z - \frac{1}{z}\right)}.$$

- b. Prove that 2mks

$$x \sin x = 2[2^2 J_2 - 4^2 J_4 + 6^2 J_6 - \dots]$$

- c. Use the Neumann series method to solve the integral equation 5mks

$$y(x) = x + \lambda \int_0^1 xz y(z) dz.$$

QUESTION FIVE

- a. Find the eigenvalues and corresponding eigenfunctions of the homogeneous Fredholm equation below 5mks

$$y(x) = \lambda \int_0^\pi \sin(x+z) y(z) dz.$$

- b. Use Schmidt–Hilbert theory to solve the integral equation 6mks

$$y(x) = \sin(x + \alpha) + \lambda \int_0^\pi \sin(x+z) y(z) dz.$$

- c. Prove the following, where $P_n(x)$ is the Legendres Polynomial

(i). $P_n(x) = 1$ 4mks

(ii). $\sum_{n=0}^{\infty} P_n(x) = \frac{1}{\sqrt{2-2x}}$ 4mks

