

CHUKA



UNIVERSITY

## UNIVERSITY EXAMINATIONS

## RESIT/SPECIAL EXAMINATION

## EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE

## ACSC 271: MATHEMATICAL METHODS FOR COMPUTER SCIENTISTS

STREAMS: BSC

TIME: 2 HOURS

DAY/DATE: WEDNESDAY 30/08/2023

8.30 A.M – 10.30 A.M.

## INSTRUCTIONS

Answer ALL Questions.

Adhere to the instructions on the answer booklet

## QUESTION ONE

- a. Sketch the graph of the function  $f(x) = \begin{cases} x-2 & \text{if } x > 0 \\ x^2+1 & \text{if } x \leq 0 \end{cases}$  and obtain  $f(-1)$  and  $f(2)$  3 marks

- b. Given that  $y = f(x) = 2x - 3$ , find  $\frac{dy}{dx}$  from first principles. 3 marks.

- c. State the order, linearity and degree of the following differential equation. 3 marks

$$x + \left(\frac{dy}{dx}\right)^2 = \frac{d^2y}{dx^2} + y^2$$

- d. Find the angle between the vectors  $\vec{a} = i + 2j + 3k$ ,  $\vec{b} = -2i + 3j + 4k$  3 marks

- e. Evaluate the following limits:

i.  $\lim_{x \rightarrow \infty} \frac{x^3 + 5x - 3}{2x^2 + 1}$  3 marks.

ii.  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$  4 marks

f. Obtain the gradient of the curve  $y = \frac{1}{x}$ , by first principles 4 marks

g. Given that  $y = \sin^{-1} \ln(3x + 4)$ , find  $\frac{dy}{dx}$  4 marks

h. Verify whether  $y = \sin 2x + 5x + 10$  is a solution of  $\frac{dy}{dx} - 2 \cos x = 5$ ,  $y(0) = 10$  3 marks

## QUESTION TWO

a. Use the trapezoidal rule with 3 strips ( $n = 3$ ) to approximate 4 marks

$$\int_1^2 (2x^2 + 5)^{\frac{3}{4}} dx$$

b. Evaluate the definite integral  $\int_0^1 12x^3 \sqrt{3x^4 + 4} dx$  4 marks.

c. Evaluate the area of the triangle with the following vertices  $A(2, 3, 1), B(4, 0, 6), C(3, 2, 4)$  4 marks

d. Prove the divergence of the series  $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$  by the ratio test 4 marks

c. Find the equation of the tangent to the curve  $y^2 = e^{2x+1}$  at the point  $x = 0$ . 4 marks.

**QUESTION THREE**

- a. Given the integral  $\int_1^2 \frac{dx}{2x}$
- (i) Use the Simpsonsrule with  $n = 5$  to approximate the integral 5 marks
  - (ii) Obtain the actual error and the error bound in the Simpsons rule. 3 marks
- b. Verify whether the differential equation below is exact hence Solve it.  
 $(4x^3y^3 + 3x^2)dx + (3x^4y^2 + 6y^2)dy = 0$  4 marks
- c. Find the equations of lines tangent to the curve  $y^2 - 6x^2 + 4y + 6 = 0$  . at the point  $y = 0$  4 marks
- d. Find the area of the parallelogram formed by the vectors  $A = 3i + j - 2k, B = i - 3j - 2k$  4 marks
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