

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF
SCIENCE IN ACTUARIAL SCIENCE

ACMT 302: ACTUARIAL MATHEMATICS II

STREAMS: BSC. ACMT

TIME: 2 HOURS

DAY/DATE: MONDAY 14/04/2025

2.30 P.M. – 4.30 P.M.

INSTRUCTIONS**Question One**

- a) What is a "state transition diagram" in multi-state models? (4 marks)
- b) Discuss the impact of an aging population on pension systems. How do emerging costs related to aging populations create financial pressures? (6 marks)
- c) You are given the following information from a life table: $l_{80} = 100$, $l_{81} = 80$, $l_{82} = 40$, $l_{83} = 20$. Find
- $p_{80:81}$ and ${}_2p_{80:81}$, (6 marks)
 - $\overline{p}_{80:81}$ and ${}_2\overline{p}_{80:81}$ (8 marks)
- d) On 1 January 2022, a life insurance company sold a number of 10-year pure endowment policies, each with a benefit amount of Kshs. 40,000, to lives then aged 30. Level premiums are payable annually in advance. Assuming AM92 Select mortality and 4% pa interest, Calculate the annual premium. (6 marks)

Question Two

- a) Define what is Profit testing. (2 marks)
- b) Consider a Markov process with state space $S = \{0,1,2\}$ and transition matrix, P:

$$P = \begin{matrix} & p & q & 0 \\ \begin{matrix} p \\ q \\ 0 \end{matrix} & 0.25 & 0 & 0.75 \\ \begin{matrix} 0.12 \\ 0.25 \\ 1-p \end{matrix} & & & \end{matrix}$$

- (i) Determine the values of p and q . (3 Marks)
- (ii) Calculate the transition probabilities $P_{ij}^{(3)}$. (4 Marks)
- (iii) Draw the transition graph for the process represented by P . (3 Marks)
- c) Explain four Important assumptions which impact the valuation of the present value of pension liabilities. (8 marks)

Question Three

- a) A life insurance company uses the three-state healthy-sick-dead model described above to calculate premiums for a 3-year sickness policy issued to healthy policyholders aged 30. Let S_t denote the state occupied by the policyholder at age $30 + t$, so that $S_0 = H$ and $S_t = H, S$ or D for $t = 1, 2, 3$.

The transition probabilities used by the insurer are defined in the following way:

$$p_{30+t}^{jk} = P(S_{t+1} = k | S_t = j)$$

For $t = 0, 1, 2$, it is assumed that:

$$p_{30+t}^{[HH]} = 0.75, \quad p_{60+t}^{[HS]} = 0.05, \quad p_{60+t}^{[SH]} = 0.65, \quad p_{60+t}^{[SS]} = 0.15$$

Calculate the probability that a new policyholder is sick at exact age 32. (3 Marks)

- b) In the context of profit-testing, explain the difference between the “profit vector” and the “profit signature”. (4 marks)
- c) A pension plan has three possible decrements: death, disability, and retirement. The following force of decrements (in terms of rates per year) apply to an individual aged 50:
- Force of death, $\mu^d(50)=0.02$,
 - Force of disability, $\mu^{di}(50)=0.01$,
 - Force of retirement, $\mu^r(50)=0.03$.

Calculate the total force of decrement at age 50 and the probability that an individual will exit the pension plan due to each of the three decrements. (7 marks)

- d) An annuity pays Kshs. 1,000 per year for 5 years to a policyholder who is 60 years old at the start. The interest rate is 5%, and the mortality probabilities are given by $1q_{60}=0.02$, $2q_{60}=0.03$,

${}_3q_{60}=0.04$, ${}_4q_{60}=0.05$ and ${}_5q_{60}=0.06$. Calculate the present value of this annuity, allowing for mortality. (6 marks)

Question Four

a) What is long-term care insurance. (2 marks)

b) A person has a sickness insurance policy that pays a benefit of 1 unit of money for each year the individual is sick. The probability of being sick at any given age x is given by the sickness function $h(x)=0.05$. The force of interest (discount rate) is $\delta=0.03$ per year.

i) Calculate the present value of the expected sickness benefits for a healthy individual at age 0, assuming the person lives forever. (6 marks)

ii) Suppose the same individual has a mortality rate of $\mu(x)=0.01$ per year, in addition to the sickness probability of $h(x)=0.05$. Calculate the present value of the expected sickness benefits, assuming a force of interest of $\delta=0.03$. (6 marks)

c) An insurance policyholder makes annual payments of Kshs. 1,000 for 4 years. The interest rate is 5% per year. The policyholder is 40 years old, and the mortality probabilities are given as follows:

- ${}_1q_{40}=0.01$ (Probability of death in the first year)
- ${}_2q_{40}=0.02$ (Probability of death in the second year)
- ${}_3q_{40}=0.03$ (Probability of death in the third year)
- ${}_4q_{40}=0.04$ (Probability of death in the fourth year)

compute the accumulated value of these payments at the end of year 4, accounting for the possibility of death during the years. (6 marks)

Question Five

- a) A life insurance company issues 20-year temporary assurance policies to lives aged 45. The sum assured, which is payable immediately on death, is £400,000 for the first 10 years, and £100,000 thereafter. Level annual premiums are payable in advance for 20 years, or until earlier death.

The premium basis is:

Mortality: AM92 Ultimate

Interest: 4% per annum

Expenses: nil.

Show that the premium payable is approximately £870.25 per annum. (9 marks)

b) An endowment policy with annual profit signatures over 6 years is given by:

$$Pr' = (-700.00, 250.00, 350.00, 450.00, 550.00, 650.00)$$

Where Pr_0 is the initial cost of the policy, and Pr_t (for $t=1, 2, \dots, 6$) represents the profit at time t .

The survival model is given by $q_{x+t} = 0.006 + 0.0004t$.

- i) Calculate the NPV for this policy using a risk discount rate of 9% per year. (4 marks)
- ii) Calculate the NPV for this policy using a risk discount rate of 14% per year. (4 marks)
- iii) Calculate the IRR for this policy. (3 marks)

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