

# FIRST YEAR UNIVERSITY RESIT EXAMINATION FOR DEGREE OF BACHELOR OF COMMERCE 

## BCOM 162: BUSINESS MATHEMATICS II

STREAMS: BCOM Y1S2
TIME: 2

## HOURS

DAY/DATE: WEDNESDAY 18/11/2020
2.30 P.M - 4.30 P.M.

## INSTRUCTIONS:

- Answer Question ONE and any other TWO Questions


## QUESTION ONE (30 MARKS)

(a) Explain the meaning of the following probability terms
(i) Mutually exclusive events
(ii) Independent events
(iii) Equally likely events
(b) Given function $y=6 X^{2}+2 X+3$

Evaluate
(i) $\frac{d^{2} y}{d X^{2}}$
(2 marks)
(ii) $\int_{0}^{3} y d X$
(c) A firm considers production of either product $\mathrm{A}, \mathrm{B}$ or C to be strategies while the likely demands for the products are considered as states of nature. The payoffs (sh.000) associated with the products are given in the table below:

|  | Strategies |  |  |
| :--- | :--- | :--- | :--- |
| Demand | A | B | C |
| High | 700 | 500 | 300 |
| Moderate | 300 | 450 | 300 |
| Low | 150 | 0 | 300 |

Use the table to determine the best production strategy using the following criteria
(i) Maximin
(3 marks)
(ii) Laplace (3 marks)
(iii) Hurwicz (Take $\alpha=0.75$ )
(3 marks)
(iv) Suppose the states of nature are believed to occur with probabilities $0.3,0.5$ and 0.2 for Low, Medium and High respectively, which product should the firm produce?
(3 marks)
(a) Use matrix algebra to solve the following system of simultaneous equations
$x+y+z=3$
$x+2 y+2 z=5$

$$
\begin{align*}
& 3 x+4 y+4 z=12 \\
& \text { marks) }
\end{align*}
$$

## QUESTION TWO (20 MARKS)

(a) Explain the meaning of decision theory and describe the decision making environments
marks)
(b) The total profit (P) per acre on a wheat farm, has been found to be related to the expenditure per acre for (a) Labour and (b) soil improvement. If $X$ represents the shillings per acre spent on labour and $Y$ represents the shillings per acre spent on soil improvement:

$$
P=48 X+60 Y+10 X Y-10 X^{2}+6 Y^{2}
$$

Required:
(i) The total profit when $X=$ Sh.1,000 and $Y=$ sh. 5,000
(ii) $\frac{\partial P}{\partial X}$
(iii) $\frac{\partial P}{\partial Y}$
(c) An economy has two industries T1 and T2. The industries have the following technology matrix $A=\left(\begin{array}{ll}0.1 & 0.2 \\ 0.2 & 0.4\end{array}\right)$

If the gross production matrix $\mathrm{X}=\binom{64}{138}$ Find the final demand matrix $\mathrm{D}=\binom{d 1}{d 2} \quad$ (8 marks)

## QUESTION THREE (20 MARKS)

(a) Find the derivative of the function $y=\frac{x^{2}-3 x}{3 x} \quad$ (4 marks)
(b) A firm has analyzed their operating conditions, prices and costs and has developed the following functions: Revenue: $\mathrm{R}=400 Q-4 Q^{2}$ and Marginal Cost: $\frac{d C}{d Q}=2 Q+10$ where Q is the number of units sold. The firm will incur a total cost of Sh. 30 when there is zero production. The firm wishes to maximize profit. Assume that all output produced is sold.
(i) What quantity should be sold?
(2 marks)
(ii) What will be the amount of maximum profit?
(c) A car manufacturer has two factories A and B assembling identical cars. It is estimated that $5 \%$ of the cars assembled in factory A and $10 \%$ of those assembled in factory B are defective. All defective cars must be reworked at a significant increase in cost. During the last eight-hour shift, factory A produced 200 cars while factory B produced 300 cars.
(i) Represent the information using a tree diagram
(2 marks)
One car is selected at random from the 500 produced;
(ii) If it is found to be defective, find the probability that it was assembled in factory A (3 marks)
(iii) Find the probability that the car is defective given that it was assembled in factory B
marks)
(d) A company produces three products $\mathrm{P}, \mathrm{Q}$ and R using raw materials $\mathrm{A}, \mathrm{B}$ and C . One unit of $P$ requires 1,2 and 3 units of $A, B$ and $C$ respectively. One unit of $Q$ requires 2,3 and 2 units of $\mathrm{A}, \mathrm{B}$ and C respectively and one unit of R requires 1,2 and 2 units of $\mathrm{A}, \mathrm{B}$ and C respectively. The number of units available for raw material $\mathrm{A}, \mathrm{B}$ and C are 8,14 and 13 units respectively. Let $\mathrm{X}, \mathrm{Y}$ and Z reprsent the number of units of $\mathrm{P}, \mathrm{Q}$ and R produced respectively. Write three matrix equations to represent the problem above.

## QUESTION FOUR (20 MARKS)

(a) Outline four areas of application of business mathematics in management (4 marks)
(b) The total cost (TC) and total revenue (TR) functions for a particular product are: $T C=2 Q+1000 T R=300 Q-2 Q^{2}$ where Q is the quantity produced and sold.

Obtain:

| i. | The average cost function | $(2$ marks $)$ |
| ---: | :--- | ---: |
| ii. | Marginal revenue function | $(2$ marks $)$ |
| iii. | Profit function | $(2$ marks $)$ |
| iv. | The quantity level at which profit will be maximum | $(2$ marks $)$ |

(c) A milk processing factory is experiencing low output and as a result, the firm's management is considering three courses of action: To arrange for subcontracting ( $\mathrm{S}_{1}$ ), to begin overtime production $\left(\mathrm{S}_{2}\right)$, or to construct new facilities $\left(\mathrm{S}_{3}\right)$. Subcontracting will cost the firm Ksh.15, 000; Overtime production will cost Ksh.33, 000 while construction of new facilities will require investment of Ksh.58, 000. The correct choice depends largely upon the future demand, which may be low, medium, or high. By consensus, management ranks the respective demand probabilities as $0.10,0.50$ and 0.40 . A cost analysis reveals effect upon the profits (in Ksh.) as shown in the table below:

| Demand | Course of action |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{S}_{1}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ |
| Low (L) | 10,000 | $-20,000$ | 150,000 |
| Medium (M) | 50,000 | 60,000 | 20,000 |
| High (H) | 50,000 | 100,000 | 200,000 |

Show this decision situation in the form of a decision tree and indicate the most preferred decision and its corresponding expected net monetary value.
(8 marks)

