

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF
BACHELOR OF EDUCATION ARTS

MATH 121: CALCULUS I

STREAMS: BED (ARTS)

TIME: 2 HOURS

DAY/DATE: TUESDAY 21/04/2020

11.30 AM – 1.30 PM

INSTRUCTIONS:

Answer Question One and any other Two Questions

QUESTION ONE (COMPULSORY) – 30 MARKS

- (a) Determine the domain of the functions $f(x) = \sqrt{4 + 3x - x^2}$ [3 marks]
- (b) Discuss the continuity of the functions $f(x) = \frac{x^2 - 81}{x + 9}$ [3 marks]
- (c) Evaluate the limits
- (i) $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x - 1}$ [3 marks]
- (ii) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$ [3 marks]
- (d) Differentiate the following functions with respect to x .
- (i) $y = \frac{x^2 + 1}{x^3 - 3x}$ [3 marks]
- (ii) $y = \sin x^2$ [3 marks]
- (e) Locate the stationery point of the curve $y = \ln x - x$ [2 marks]
- (f) Given $x = \sin t$ and $y = \cos t$, determine $\frac{dy}{dx}$ [2 marks]

- (g) Given $y \cos x = e^x$ show that $\frac{d^2y}{dx^2} - 2 \tan x \frac{dy}{dx} - 2y = 0$ [4 marks]
- (h) Determine the slope of the tangent line to the graph $x^2 + 4y^2 = 4$ at the point $(\sqrt{2}, -\frac{1}{\sqrt{2}})$ [4 marks]

QUESTION TWO (20 MARKS)

- (a) Evaluate $\lim_{x \rightarrow \infty} \frac{\sqrt[5]{x^3 - 7x + 1}}{\sqrt{32x^3 + 5x^2 - 7}}$ [3 marks]
- (b) A spherical balloon in being blown up so that its volume increases at the rate of 2cm^3 per second. Find the rate at which the radius increases when the volume of the balloon is 100cm^3 [5 marks]
- (c) Differentiate the following functions with respect to x :
- (i) $y = x^{x^2}$ [4 marks]
- (ii) $y = \sin^{-1}(\ln x)$ [4 marks]
- (iii) $y = (3x - 2x^2)^3(5 + 4x)$ [4 marks]

QUESTION THREE (20 MARKS)

- (a) Determine all the turning points of the curve $y = 5 + 24x - 9x^2 - 2x^3$ and sketch the curve. [5 marks]
- (b) Determine the coordinates of the points on the curve $y = x^3 - 6x^2 + 12x + 2$ at which the tangent is parallel to the line $y = 3x$. [5 marks]
- (c) Given $y^3 + xy^2 - 5x + x^2 = 5$, determine $\frac{dy}{dx}$ [3 marks]
- (d) Given that $f(x) = 5 - \frac{4}{x}$, find all the values of C in the open interval $(1, 4)$ such that $f'(c) = \frac{f(4) - f(1)}{4 - 1}$ [3 marks]
- (e) Evaluate $\lim_{x \rightarrow \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$ [4 marks]

QUESTION FOUR (20 MARKS)

- (a) Show that $\frac{d}{dx}(\sin x) = \cos x$ [5 marks]
- (b) Sketch the graph of $f(x)$ with the following characteristics
- $$f(2) = f(4) = 0$$

$$f'(x) < 0 \text{ if } x < 3$$

$$f'(3) \text{ is undefined}$$

$$f'(x) > 0 \text{ if } x > 3$$

$$f''(x) < 0 \text{ if } x \neq 0$$

[3 marks]

- (c) Find $f'(x)$ given $f(x) = x^4 \sin x^3$ [4 marks]
- (d) Given $f(x) = x^3 + 3x^2 + 2$, determine the interval where $f(x)$ is increasing and decreasing. [4 marks]
- (e) Use the method of first principles to determine $\frac{dy}{dx}$ given $y = \frac{2}{x}$ [2 marks]

QUESTION FIVE (20 MARKS)

- (a) Sketch the intersecting graphs of the equations $2x^2 + y^2 = 6$ and $y^2 = 4x$ and show that they are orthogonal. [6 marks]
- (b) Given $y = \frac{f(x)}{g(x)}$, $g(x) \neq 0$, where $f(x)$ and $g(x)$ are differentiable functions of x , show that $\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ [5 marks]
- (c) Approximate $\ln 1.1$ given $\ln 1 = 0$ [3 marks]
- (d) Find the equation of the normal and the tangent to the curve $y = \frac{4}{x}$ at the point $x = 1$ [4 marks]
- (e) Determine $\frac{dy}{dx}$ given $y = 2^x$ [2 marks]
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