

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF
MASTERS OF SCIENCE IN APPLIED MATHEMATICS

MATH 809: COMPLEX ANALYSIS

STREAMS: MSC (APP MATH)

TIME: 3 HOURS

DAY/DATE: TUESDAY 14/04/2020

11.30 AM – 2.30 PM

INSTRUCTIONS:

- Answer any Three Questions
- Use advanced scientific calculators

QUESTION ONE (20 MARKS)

(a) Find all the complex roots of the following equations

(i) $Z^6 = -9$ [8 marks]

(ii) $Z^2 + 2Z + (1 - i) = 0$ [5 marks]

(b) Find the Laurent series of $f(z)$ in the given domain $f(z) = \frac{z^2}{z^2 - 3z + 2}$ in the domain $1 \leq |z| \leq 2$ [7 marks]

QUESTION TWO (20 MARKS)

(a) Let c be the circle $|z| = 1$ oriented clockwise.

(i) Compute $\int_c \frac{1}{z^2 - 8z + 1} dz$ [5 marks]

(ii) Given that $Z = e^{i\theta}$ show that $\int \frac{1}{z^2 - 8z + 1} dz = \int_{-\pi}^{\pi} \frac{1}{4 - \cos \theta} d\theta$ and hence compute $\int_0^{\pi} \frac{1}{4 - \cos \theta} d\theta$ [9 marks]

(b) Find all the complex roots of the following equations

(i) $\bar{z} = z$ [3 marks]

(ii) $z \bar{z} = 9$ [3 marks]

QUESTION THREE (20 MARKS)

(a) (i) Explain the meaning of a conformal mapping. [2 marks]

(ii) Consider the transformation $w = (1 + i)z$ on the rectangle whose corner points are $(1, 0)$, $(1, i)$, $(0, i)$ and $(0, 0)$. Sketch the diagram and describe the transformation. [4 marks]

(b) Find the Mobius map that maps the points

$z_2 = 2, z_3 = i, z_4 = -2$ onto $w_2 = 1, w_3 = i$ and $w_4 = -1$ respectively. [6 marks]

(c) Verify that $u(x, y)$ is harmonic and find the conjugate harmonic function $v(x, y)$ given that $u(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$ and $v(x, y) = u + i v$ [8 marks]

QUESTION FOUR (20 MARKS)

(a) (i) State the Cauchy integral theorem and Cauchy integral formula. [4 marks]

(ii) Hence evaluate the integral using Cauchy's integral formula.

$$\int_C \frac{2dz}{z^2-1}$$
 [8 marks]

(b) (i) Show that $\lim_{z \rightarrow \infty} \frac{4z^5}{z^5-42z} = 4$ [4 marks]

(ii) Provided that $z \neq -\frac{1}{2}$, find $f'(z)$ given that $f(z) = \frac{z+1}{2z+1}$ [4 marks]

QUESTION FIVE (20 MARKS)

(a) Consider the functions $f(z) = \frac{1}{z^4+5z^2+6}$ find,

(i) The poles [4 marks]

(ii) The residues [8 marks]

(b) Graph the region $\{z: |z - 4i + 2| > 2\}$ in the complex plane [3 marks]

(c) Compute $\lim_{z \rightarrow i} \frac{iz^3+1}{z^2+1}$ [5 marks]