

MATH 125 – Discrete Mathematics

Instructions

1. Answer question **ONE** and any **OTHER TWO** questions from the remaining **FOUR**
2. Show all your workings on the answer booklet provided.
3. Marks will be awarded for the correct working even if the answer is wrong.
4. Marks for each Part of a question are as shown.
5. Start a new question on a fresh page.

QUESTION ONE – 30 MARKS

a) Let $*$ be a binary operation on the set of integers, defined by

$a * b = a + b - ab$ for every $a, b \in \mathbb{Z}$. Determine whether or not $*$ is

- i. Commutative
- ii. Associative
- iii. Find an identity element with respect to $*$ if it exists

5 Marks.

- b) Find the product of the polynomials $f(x) = 4x^3 - 2x^2 + 3x - 1$ and $g(x) = 3x^2 - x - 4$ over Z_4 **4 Marks.**
- c) By use of examples, differentiate partial and total ordering **4 Marks.**
- d) Determine the validity of the following argument
 S_1 : Thieves are jailed
 S_2 : Serious people read good books
 S_3 : Graduates are serious people
Conclusion: No graduate is thief **3 Marks.**
- e) Let A, B and C be three finite sets with $|A| = 8$, $|B| = 10$, $|C| = 8$, $|A \cap B| = 5$, $|A \cap C| = 4$, $|B \cap C| = 7$ and $|A \cup B \cup C| = 13$. Find $|A \cap B \cap C|$. **4 Marks.**
- f) Find the product of the polynomials $f(x) = 7x^3 - 4x^2 + 3x - 11$ and $g(x) = 15x^3 + 3x^2 - x - 14$ over Z_5 **4 Marks**
- e) Let R be a relation defined on the set $A = \{0,1,2,3\}$ containing the ordered pairs (0,1), (1,1),(1,2),(2,0),(2,2) and (3,0). Determine
- The reflexive closure of R
 - The symmetric closure of R **4 Marks**
- f) Translate the logical equivalence $(T \wedge T) \vee \neg F = T$ into an identity in Boolean algebra **2 Marks**

QUESTION TWO – 20 MARKS.

- a) Given the propositional variables,
 p : The day was June 23rd 2020
 q : Four out of six patients tested positive of Covid.

r : Two patients succumbed to Covid(died).

Write the following statements in terms of p , q , r and the logical connectives.

i) The day was *June 23rd 2020* and the four patients out the six were confirmed positive of Covid.

ii) If two patients died, then the day *June 23rd 2020* and the hospital confirmed the four out of the six tested positive of Covid.

iii) Represent the statement in ii) above using their respective propositional variables. Construct a truth table for the statement. State with reasons the type of compound statement this is.

8 Marks.

b) Show that $(p \rightarrow q) \equiv \{(\sim p) \vee q\}$

2 Marks

a) Prove the DeMorgan's law in Boolean algebra $(x + y)' = x' y'$

5 Marks

b) d. (i) Explain the following terminologies as used in set theory

- Power set
- Set cardinality

(ii) Find $P(A)$ of $A = \{ (a,b), (c), (d,f,g) \}$

5 Marks.

QUESTION THREE – 20 MARKS

a) Use Euclidean Algorithm to find the gcd of 711 and 663, and express it as a linear combination of 711 and 663.

5 Marks

(ii) Find all the integers x which are a solution to the following congruence relation $663x \equiv 6 \pmod{711}$

5 Marks

b) Let $Z = \{ x / x \text{ is an integer} \}$, Constitute two subsets A_1 and A_2 such that they are a partition of Z .

4 Marks

c) Prove by mathematical induction that

$$1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2 = \frac{n(n+1)(n+2)}{6}$$

6 Marks

QUESTION FOUR – 20 MARKS.

a) a) Consider the third order homogeneous recurrence relation $a_n = 2a_{n-1} + 3a_{n-2}$

i. Find the general solution

5 Marks

ii. Find the initial solution given $a_0 = 1, a_1 = 3$,

5 Marks

marks)

c. What are Bell Numbers

Let $S = \{1, 2, 3\}$ find the Bell Number of Set S .

5 marks

d) Given that $f(x) = \frac{1}{3}x + \frac{2}{3}$ and $g(x) = 3x - 2$, find

i) $(g \circ f)(x)$

ii) $(f \circ g)(x)$

5 Marks.

QUESTION FIVE - 20 MARKS

a) Simplify the Boolean expression $(\overline{A \bullet B}) + (\overline{A + B})$ by using De Morgan's laws and the rules of Boolean algebra.

4 Marks

b) With the aid of a diagrammatic explanation, device a logic system that meets the requirements of $(\overline{P + Q}) \bullet (\overline{R + S})$

5 Marks

a) Use the principle of mathematical induction to prove that

$$1^2 + 3^2 + 5^2 + \dots + (2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$$

6 Marks.

c) Differentiate between a binary operator and a postulates

5 Marks

Good Luck