CHUKA   **UNIVERSITY**

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN APPLIED COMPUTER SCIENCE

ACSC 271: MATH METHODS FOR COMPUTER SCIENTISTS

STREAMS: **TIME: 2 HOURS**

DAY/DATE: TUESDAY 11/12/2018   **2.30 P.M. – 4.30 P.M.**

INSTRUCTIONS:
- **Answer question ONE and any other TWO questions.**
- **Adhere to the instructions on the answer booklet.**
- **Do not write on the question paper.**

QUESTION ONE (30 MARKS)

(a)   Find the domain for the function

$$f(x) = \sqrt{x^2 - x - 6}$$ (4 marks)

(b)   Evaluate the following limit

$$\lim_{x \to \infty} \frac{4x^2 + 10}{x^2 - 4x}$$ (3 marks)

(c)   Obtain $\dfrac{dy}{dx}$ of the function $f(x) ¿ \cos^{-1}(2x + 3)$ (4 marks)

(d)   Find the equation of the tangent to the curve $x^2 + y^2 = 5$ at the point $x = 1$ (4 marks)

(e)   Solve the differentiate equation

$$\frac{dy}{dx} = \sqrt{x} + 3x^2, \quad \text{Given} \quad y(0) = 3$$ (4 marks)

(f)   Solve the simultaneous equation

$$2x+y=4$$
$$2x-y=0, \quad \text{by row reduction}$$ (3 marks)

(g)   Find the value of t for which the vectors

$$\tilde{a} = ti-5j+2k \quad \text{and} \quad \tilde{b}=i+4j-k \quad \text{are orthogonal}$$ (4 marks)

(h)   Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{n!} \quad \text{Using the ratio test.}$$ (4 marks)

QUESTION TWO

(a)   Evaluate the limit of the function.

$$\lim_{x \to 0} \frac{\sin 3x}{\sin x}$$ (4 marks)

(b)   Find the derivative for the functions below

(i) $f(x)=\dfrac{1}{x+1}$ by first principles   (4marks)

(ii) $f(x)=\sin(3x^2+5)$ (4marks)

(c)   Prove the series

$$\sum_{n=1}^{\infty} \frac{2n¿^{!}}{n!¿^{2}}$$ is divergent   (8 marks)

QUESTION THREE

(a)   Consider matrix A where

$$A=\begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$ Use the Layley Hamilton theorem to obtain A^{-1} (7 marks)

(b)   Find all the given values and eigen vectors of matrix B below

$$B=\begin{pmatrix} -2 & +2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$ (10 marks)

(c)   Find the angle between two vectors

$$\tilde{a}=i-2j+4k$$
$$\check{b}=-4i+j-2k$$ (3 marks)

QUESTION FOUR

(a)   Solve the differential equation $\dfrac{dy}{dx}-2y=x$ using a suitable integrating factors. (5 marks)

(b)   Obtain the actual error in the evaluation of

$$\int_1^2 \frac{dx}{x}$$ using the trapezoidal rule to 4 d p.   (8 marks)

(c)   Discuss the consistency of the following system of equations using row reduction method hence solve it if found consistent.

$$2x+3y+4z=11$$

$$x+5y+7z=15$$

$$3x+11y+13z=25$$ (7 marks)

QUESTION FIVE

(a)   Show that the differential equation
$$\left(5x^4+3x^2y^2-2xy^3\right)dx+\left(2x^3y-3x^2y^2-5y^4\right)dy=0$$
is exact hence solve it.   (5 marks)

(b)   Prove the convergence of the series $\sum_{n=1}^{\infty} \dfrac{n}{3^n}$ using the root test.   (3 marks)

(c)   Find the magnitude of vector orthogonal to the vectors $\tilde{a} \vee \tilde{b}$ given

$$\tilde{a}=i+3\,j-k,\tilde{b}=2\,i-j+k$$ (4 marks)

(d)   Evaluate the limit

$$\lim_{x \to 3} \frac{6\,x^2-10\,x-24}{2x-6}$$
(3 marks)

(e)   Obtain the inverse of the function $f(x)=2\,x^2+5$ (2 marks)

(f)   Obtain the gradient of the curve $y=x^x$ at the point $x=1$ (3 marks)