

CHUKA



UNIVERSITY

**SUPPLEMENTARY / SPECIAL EXAMINATIONS**

**FOURTH YEAR EXAMINATION FOR THE AWARD OF BACHELOR DEGREE IN**

**ECON 234/220: MATHEMATICS FOR ECONOMISTS II**

**STREAMS:**

**TIME: 2**

**HOURS**

**DAY/DATE: TUESDAY 17/11/2020**

**8.30 A.M - 10.30 A.M.**

**INSTRUCTIONS:**

- Answer Question One and any other Two Questions

Question One

a). Given the following utility function

$$U = f(x, y) = -2 + xy + 40x + 2y$$

- (i). Find the critical values of x and y and the stationary point (3 marks)
- (ii). Confirm that the stationary point gives rise to maximum utility (2 marks)

b). Consider the following constrained optimization problem

$$\text{Optimize } Z = 2 + 2xy + 4$$

$$\text{Subject to } 4x + 4y = 32$$

- (i). Find the critical value of x, y and (4 marks)
- (ii). Confirm the existence of a minimum or a maximum (3 marks)

c). Solve for

$$\ln ( = 2 \quad (3 \text{ marks})$$

d). What rate of growth will double GNP in twenty years given that the growing value of GNP is dictated by the following function?

$$= \quad (3 \text{ marks})$$

e). Given the following differential equation and the accompanying initial conditions

$$+ 3y = 12$$

$$T=0 \text{ and } y = 24$$

Find

- (i). The homogenous solution (3 marks)

- (ii). The particular solution (2 marks)
  - (iii). The general solution (1 mark)
  - (iv). The unique and definite solution (2 marks)
- f). (i) Find consumer's surplus, given the following supply functions and equilibrium price  
 $P = 3 + 2Q = 9$  (3 marks)
- (ii). Draw a sketch to show this surplus (1 mark)

Question Two

a). A firm wishing to maximize its output subject to budget (cost) constraint has the following production function and cost constraint

$$\begin{array}{ll} \text{Production function} & Q = \\ \text{Cost function} & 2K + 4L = 40 \end{array}$$

- (i). Set up constrained maximization problem (1 mark)
- (ii). Form the Lagrangian function (1 mark)
- (iii). Find the critical values of x, y and (5 marks)
- (iv). Confirm the presence of a maximum (2 marks)

b). Solve the following linear programming problem graphically and give the feasible point

$$\begin{array}{ll} \text{Maximize} & TR = 0.6X + 0.4Y \\ \text{subject to} & X + Y \leq 1000 \\ & X + 2Y \leq 1600 \\ & 2X + Y \leq 1600 \end{array} \quad (6 \text{ marks})$$

c). Find the second derivative of the following  
 $y =$  (2 marks)

d). Prove that  
 $= + +$  (3 marks)

Question Three

- a). Differentiate between Hessian matrix and Bordered Hessian matrix (4 marks)
- b). A farmer growing trees of timber knows the growing value of his tree is defined by the following exponential function  $V = 40e^{0.06t}$ . Assuming zero maintenance and planting costs, how long should the farmer grow the trees before cutting them for sale if his intention is to maximize profits derived from such sale, given a discount rate of 6 per cent under continuous compounding (8 marks)

c). Define the term optimal timing and give its importance in mathematical economics (5 marks)

d). Given the following primal problem

$$\begin{array}{ll} \text{Maximize revenue} & 12x + 15y \\ \text{Subject to} & 12x + 12y \leq 48 \\ & 6x + 12y \leq 72 \\ & 14x + 12y \leq 84 \end{array}$$

$$x \geq 0; y \geq 0$$

Get the dual problem (3 marks)

Question Four

a). Explain three different types of linear programming problems (3 marks)

b). Find the value of Q that satisfies the first order condition for maximization (3 marks)

c). Given the following information

$$= + \quad =$$

Where

(i). Consumption expenditure is a function of permanent income

$$= + \quad > 0; 0 << 1$$

$$= +$$

(ii). Permanent income at time t is equal to weighted average of  $= a + (1 - a)$

Find:

(i). The homogenous solution (2 marks)

(ii). Particular solution (1 mark)

(iii). The general solution (1 mark)

(iv). The unique solution (1 mark)

(v). Discuss the convergence properties of the system, showing clearly what the steady state value is. Provide graphical sketch of the time path of (5 marks)

.....