THARAKA UNIVERSITY COLLEGE

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR

MATH 122: BASIC MATHEMATICS

STREAMS: P/T **TIME: 2 HOURS**

DAY/DATE: MONDAY 06/04/2020 **8.30 A.M. – 10.30 A.M.**

INSTRUCTIONS: Answer question ONE and any other TWO questions

QUESTION ONE (30 MARKS)

(a) Define

(i) Set [1 mark]

(ii) Tautology [1 mark]

(iii) Proposition [1 mark]

(b) Construct a truth table for $P \wedge R \Leftrightarrow \sim P \vee Q$ [4 marks]

(c) Let $f: A \longrightarrow B$ where $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$. Determine whether f is a bisection given that $F(1) = a$, $F(2) = c$, $F(3) = d$ and $F(4) = a$ [4 marks]

(d) Given that $3n + 2$ is odd. Prove that n is odd [5 marks]

(e) Given that $f(x) = \frac{3x^2+12}{4x+1}$. Find $f(3)$ [2 marks]

(f) If a club has 20 members, how many different four member committee are possible [4 marks]

(g) Prove that $\cos\theta\,(\tan\theta - \sec\theta) = sin\theta - 1$ [4 marks]

(h) If the third term of a geometric progression is the square of the first term and the fifth term is 64. Find the first term and common ratio [4 marks]

QUESTION TWO

(a) Let $U = \{w, y, x, z, 2, 4, 6\}$ be a universal set

Set $A = \{w, x, 2\}$, $B = \{z, 4, 6\}$. Find

(i) $(A\ U\ B)^c$ [2 marks]

(ii) $A^c \cup B^c$ [2 marks]

(iii) $(A - B)\ U\ (B - A)$ [2 marks]

(b) Use mathematical induction to prove that $1 + 3 + 5 + .. + (2n - 1) = n^2$ for all $n \in \mu$

[5 marks]

(c) Prove that if n is odd, then n^3 is odd [5 marks]

(d) Define the terms

(i) Valid argument [2 marks]

(ii) Logical equivalence [2 marks]

QUESTION THREE (20 MARKS)

(a) Given that $f(x) = 3x$, $g(x) = 2x + 4$ and $h(x) = {}^1\!/_4\, x$

Find

(i) $(fogoh)x$ [2 marks]

(ii) $(gofoh)x$ [3 marks]

(iii) $g^{-1}(x)$ [3 marks]

(b) Negate the statement “every student in class has done cat I [2 marks]

(c) Check whether $(P\ \wedge q)\ V \sim r \rightarrow q \leftrightarrow r$ is a tantology or not [5 marks]

(d) Given that $A = x, y, t$ and $B = \{a, b, c\}$, show that $AXB \neq BXA$ [5 marks]

QUESTION FOUR

(a) With the inverse and converse of the statement, “If you are a registered student, then you can access library services in campus”. [2 marks]

(b) Determine whether the function $f(x) = x^2$ from Z to z is one to one. [3 marks]

(c) Find the modulus and conjugate of

$Z = \frac{2-i}{\sqrt{2}+4i}$ [5 marks]

(d) Prove that $\sqrt{2}$ is irrational by contradiction [5 marks]

(e) Find the first three terms of the sequence $a_n = (1+i)^n$, $i = \sqrt{-1}$. Hence find the sum of the first three terms [5 marks]

QUESTION FIVE (20 MARKS)

(a) List the members of the set $A = \{x := 2x^2 - 4x - 6 = 0\}$ [3 marks]

(b) Evaluate $\sum_{k=1}^{6} (-1)^{k+1} 2k$ [4 marks]

(c) In a survey of 60 people, it was found that

25 read the daily nation

26 read the standard

9 read both daily nation and Kenya times

11 read both nation and standard

8 read both standard and Kenya times

3 read all the three newspapers

(i) Fill the correct number of people in a Venn diagram where N, S and T denote the number of people who read nation, standard and Kenya times newspapers respectively. [8 marks]

(ii) The number of people that read at least one of the three newspapers [3 marks]

(iii) The number of people who read exactly one of the newspapers [2 marks]
