

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF DEGREE
OF BACHELOR OF COMMERCE

BCOM 162: BUSINESS MATHEMATICS II

STREAMS: BCOM (Y1S2)

TIME: 2 HOURS

DAY/DATE: WEDNESDAY 11/4/2018

2.30 P.M. – 4.30 P.M.

INSTRUCTION: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- (a) Explain the meaning of the following probability terms
- (i) Mutually exclusive events [2 marks]
 - (ii) Independent events [2 marks]
 - (iii) Equally likely events [2 marks]
- (b) Given function $y = 120X - X^2 + 0.02X$
- Evaluate
- (i) $\frac{d^2y}{dX^2}$ [2 marks]
 - (ii) Explain three uses of calculus in business. [3 marks]
- (c) A firm considers production of either product A, B and C to be strategies while the likely demands for the products are considered as states of nature. The payoffs (sh. 000) associated with the products are given in the table below:

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	Strategies		
Demand	A	B	C
High	700	500	300
Moderate	300	450	300
Low	150	0	300

Use the table to determine the best production strategy using the following criteria

- (i) Maximin [2 marks]
 - (ii) Hurwicz (Take $\alpha = 0.75$) [2 marks]
 - (iii) Laplace [2 marks]
 - (iv) Suppose the states of nature are believed to occur with probabilities 0.3, 0.5 and 0.2 for low, medium and high respectively, which product should the firm produce?
- (d) An electronic manufacturer has two lines A and B assembling identical electronic units. 5% of the units assembled on line A and 10% of those assembled on line B are defective. All defective units must be reworked at a significant increase in cost. During the last eight hour shift, line A produced 200 units while the line B produced 300 units. One unit is selected at random from the 500 units produced.
- (i) If it is found to be defective, find the probability that it was assembled in line A [3 marks]
 - (ii) Find the probability that the unit is defective given that it was assembled on line B [2 marks]
- (e) Use matrix algebra to solve the following system of simultaneous equations
- $$x + y + z = 3$$
- $$x + 2y + 2z = 5$$
- $$3x + 4y + 4z = 12$$
- [6 marks]

QUESTION TWO (20 MARKS)

(a) Explain the meaning of decision theory and describe the decision making environments. [6 marks]

(b) The total profit (P) per acre on a wheat farm, has been found to be related to the expenditure per acre for (a) labour and (b) soil improvement. If X represents the shillings per acre spent on labour and Y represents the shillings per acre spent on soil improvement:

$$P = 48X + 60Y + 10XY - 10X^2 - 6Y^2$$

Required:

(i) $\partial P / \partial X$ and $\partial P / \partial Y$ [2 marks]

(ii) What are the values of X and Y that maximize profits? [3 marks]

(c) The following system of equations represents the inter-sector demand and final demand of three sectors in an economy that has three industries T1, T2 and T3. The industries have the following technology matrix.

$$A = \begin{bmatrix} 0.2 & 0.2 & 0.1 \\ 0.2 & 0.4 & 0.1 \\ 0.1 & 0.2 & 0.3 \end{bmatrix} \text{ and } D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}. \text{ If the gross production matrix } X = \begin{bmatrix} 40 \\ 45 \\ 50 \end{bmatrix} \text{ where}$$

$d_1, d_2,$ and d_3 are the final demands in metric tonnes from sector 1, 2 and 3 respectively.

Required:

(i) Explain the meaning of closed Leontief model [1 mark]

(ii) Determine the Leontief inverse matrix $(1 - A)^{-1}$ [4 marks]

(iii) Solve for D (final demand) column vector in the matrix equation $X - AX = D$ [4 marks]

QUESTION THREE (20 MARKS)

(a) Find the integral of the function $\int_0^2 2x^4 - x^3 + 7dx$ [3 marks]

(b) A firm has analyzed their operating conditions, prices and costs and has developed the following functions: Revenue: $R = 400Q - 4Q^2$ and marginal cost: $dC/dQ = 2Q + 10$ where Q is the number of units sold.

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The firm will incur a total cost of sh. 30 when there is zero production. The firm wishes to maximize profit. Assume that all output produced is sold.

- (i) What quantity should be sold? [2 marks]
 - (ii) What will be the amount of maximum profit? [3 marks]
- (c) The following system of equations represents the inter-sector demand and final demand of two sectors in a given economy.

$$0.4X_1 + 0.2X_2 + 10 = X_1$$

$$0.3X_1 + 0.1X_2 + 12 = X_2$$

Where X_1 and X_2 are outputs in metric tonnes of sector 1 and 2 respectively while d_1 and d_2 are the final demands in metric tonnes from sector 1 and 2 respectively.

Required: Determine

- (i) The technology coefficients matrix and surplus matrix [2 marks]
 - (ii) The gross production matrix [4 marks]
- (d) A company produces three products X, Y and Z using raw materials A, B and C. One unit of X requires 1, 2 and 3 units of A, B and C respectively and one unit of Y requires 2, 3 and 1 of A, B and C respectively while Z requires 1, 2 and 2 units of A, B & C. The number of units available for raw material A, B and C are 8, 14 and 13 units respectively. Using the matrix method, find how many units of each product to produce in order to utilize completely the available resources. [6 marks]

QUESTION FOUR (20 MARKS)

- (a) Explain the importance of matrices in business. [2 marks]
- (b) A retailer of motorized bicycles has examined cost data and has determined an average cost function which expresses the annual cost of purchasing, owning, and maintaining inventory as a function of the size (number of units) of each order it places for the bicycles. The average cost of function is,

$$AC = f(q) = \frac{4,860}{q} + 15q + 750,000$$

Where AC equals average annual inventory cost, stated in dollars and q equals the number of motorcycles ordered each time the retailer replenishes the supply.

- (i) Determine the order size, which minimizes annual inventory cost. [2 marks]
- (ii) Determine the total cost function. [2 marks]

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- (iii) What is minimum total annual inventory cost expected to equal? [2 marks]
- (c) Kaanwa Agro mills ltd (KAM) is considering whether to enter a very competitive market for manufacturing of animals feeds. In case KAM decides to enter this market it must either install a new high technology equipment or pay overtime wages to the entire staff. In either case, market entry could result in:
- (i) High sales
 - (ii) Medium sales
 - (iv) Low sales
 - (v) No sales

The management of KAM has estimated that if they enter the market there is a 60% chance of their shareholders approving the installation of the new equipment (this means that there is a 40% chance of using overtime). A random analysis of the current market structure reveals that KAM has 40% chance of achieving high sales, a 30% chance of achieving medium sales, a 20% chance of achieving low sales and a 10% chance of achieving no sales.

Financial analysis of KAM indicates that a high level of sales will yield shs 1,000,000 profit, medium level of sales will result in a shs 600000 profit, a low level of sales will result in a shs 200000 profit and a no sales level will cost KAM a loss of shs 500,000. Entering the market will require a cash outlay of either shs 300,000 to purchase and install the new equipment or shs 100,000 for overtime expenses, should the second option be selected. A decision not to enter the market will add zero extra profit to the firm.

(Assume that all costs and revenues have been discounted to their present values)

- (a) Construct a decision tree for the problem showing clearly the courses of action.
- (b) By applying an appropriate decision criterion recommend whether or not KAM should enter the market [12 marks]