CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF MASTER OF SCINCE IN ECONOMICS AND MASTER OF SCIENCE IN AGRICULTURAL ECONOMICS

MSEC 832: MATHEMATICAL METHODS FOR ECONOMISTS

STREAMS: MSEC, MSAE YISI **TIME: 3 HOURS**

DAY/DATE: THURSDAY 09/08/2018 **2.30 PM – 5.30 PM**

INSTRUCTIONS:

Answer Question One and any other Three Questions from the remaining

QUESTION ONE

(a) Given the following differential equation and the accompanying initial conditions
$\acute{y} = -3y + 12t = 0 \wedge y = 24\, i.e\, y(0) = 24$

Find:

(i) Homogenous solution [2 marks]

(ii) Particular solution [2 marks]

(iii) General solution [2 marks]

(iv) Unique or definite solution [2 marks]

(b) Suppose the growing value of wine is represented by the following exponential function $V = 200e^{t^{\frac{1}{4}}}$. Considering a discount rate of 12% under continuous compounding, find the optimal time and confirm that the growing value of wine is maximized at $(\acute{t})$. [12 marks]

QUESTION TWO

(a) (i) Find the sign of Q given the following: (hint apply descriminant approach)

$$Q=[X\,Y\,Z]\begin{bmatrix}-6 & 0 & 1\\ 0 & -5 & 2\\ 1 & 2 & 4\end{bmatrix}\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}$$

1×3

$3\times 3\,3\times 1$

$Q=X^{'}D\,X$ [3 marks]

(ii) Find the sign of Q by Hessian approach. [4 marks]

(b) (i) Present the following in two forms of matrix format and then find $\frac{\partial z}{\partial x}$

$Z=3x_1+4\,x_2+6\,x_3+5\,x_4$ [2 marks]

(ii) Given the following sets of matrices

$$Q=[X_1X_2]\begin{bmatrix}4 & 3\\ 3 & 7\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

1×2

$2\times 2\,2\times 1$

$Q=X^{'}D\,X$

$1\times 2\,2\times 2\,2\times 1$

Find $\frac{\partial Q}{\partial X}$ (Hint: expand the polynomial then apply the formula $\frac{\partial q}{\partial x}=\begin{bmatrix}\frac{\partial q}{\partial x_1}\\ \frac{\partial q}{\partial x_2}\end{bmatrix}$

[3 marks]

(c) Consider the following polynomial function $Q=2\,X^2-4\,XY+6\,Y^2-8\,yz+6\,z^2$

(i) Is Q a form? [1 mark]

(ii) If it is a form, what type of form is it? [1 mark]

(iii) Present Q in matrix format. [2 marks]

(iv) Find the sign of Q by discriminant and Hessian approach. [4 marks]

QUESTION THREE

(a) A firm in a perfectly competitive market has the following demand, total variable cots and total fixed costs functions:

Demand function: $P=12.1$

Total variable cost: $TVC=\frac{1}{20}Q^{2}-1.5Q^{2}+17.5Q$

Total fixed costs = $FC=50$

Obtain the following functions:

(i) Total cost [1 mark]

(ii) Total revenue [1 mark]

(iii) Profit [4 marks]

(iv) Find the output level at which profits are maximized. [5 marks]

(v) Compare the marginal cost and marginal revenue at the profit maximizing output and comment. [5 marks]

(b) Demonstrate Euler's theorem for
$Q=AK^{\frac{2}{5}}L^{\frac{8}{5}}$ [4 marks]

QUESTION FOUR

A consumer is faced with the following utility function and budget constraint: $\cup=f(Q_1Q_2)$.

His budget M=240 and the exogenously determined prices of good $Q_1\wedge Q_2$ are respectively given as follows: $P_1=2\wedge P_2=2$

(i) Set up the consumers budget constraint [1 mark]

(ii) Set up the constrained utility maximization problem for the consumer. [2 marks]

(iii) What is the corresponding composite or Lagrangian function? [2 marks]

(iv) Determine the critical values of Q_1 and Q_2 and λ . [6 marks]

(v) By applying the second order condition confirm that the critical values present a maximum utility. [9 marks]

QUESTION FIVE

A three sector input-output table is given by the following:

	1	2	3	D	X
1	x_{11}	x_{12}	x_{13}	D_1	X_1
2	x_{21}	x_{22}	x_{23}	D_2	X_2
3	x_{31}	x_{32}	x_{33}	D_3	X_3
V	V_1	V_2	V_3		
X	X_1	X_2	X_3		

(i) The input-output coefficients are denoted by $\propto_{i'j'}(i',j'=1,2,3)$ write out these input output coefficients in terms of $X_{i'j'}(i',j'=1,2,3)$ and the total outputs [3 marks]

(ii) Find x_{12}, x_{31}, x_{33} in terms of $\propto_{i'j'} \wedge X_j$ [3 marks]

(iii) Find GNP by expenditure approach [1 mark]

(iv) Derive the input-output model [10 marks]

(v) Insert the dimensions of the different elements of the input-output model. [3 marks]

QUESTION SIX

(a) Given the following composite function: $Z = X^3 Y^2 X = \mu^3 Y = \mu^2$ FIND:

(i) ∂z [2 marks]

(ii) $\partial^2 z$ [2 marks]

(iii) $\frac{\partial z}{\partial \mu}$ and present the direct and the indirect impact of U and Z. [2 marks]

(b) By applying total differentiation and implicit function differentiation, find $\frac{\partial y}{\partial x}$ given the following function $4X^3 Y^4 + 4Y^2 + 83 = 0$ [5 marks]

(c) A utility function is given as $V_0 = X^{\frac{1}{5}} Y^{\frac{3}{5}}$

(i) By applying differentials and implicit function differentiation, find $MRCS_{YX}$ given $V_0 = 2 \wedge X = 2$ [6 marks]

(ii) Interpret your results. [3 marks]
