

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF SCIENCE IN
ECONOMICS

MSEC 832: MATHEMATICAL METHODS FOR ECONOMICS

STREAMS: MSC

TIME: 3 HOURS

DAY/DATE: MONDAY 23/07/2018

2.30 P.M. – 5.30 P.M.

INSTRUCTIONS:

ANSWER ANY THREE QUESTIONS.

QUESTION ONE (20 MARKS)

- (a) The average revenue and average cost function for a competitive firm are given as follows:

$$AR = 4 - \frac{1}{4}Q$$

$$AC = \frac{4}{Q} + 2 - 0.3Q + 0.05Q^2$$

Find the level of output Q and price P that will maximize profits of the firm. (6 marks)

- (b) Given the production function as $Q = 15K^{\frac{2}{5}}L^{\frac{3}{5}}$; where Q is the output and L and K are labor and capital respectively;

- (i) Find the marginal product of labor and marginal product of capital. (6 marks)
- (ii) Determine whether or not the function is characterised with diminishing returns to a factor. (3 marks)

- (c) Solve for x in the equation: $e^{2x-100}=150$. (5 marks)

QUESTION TWO (20 MARKS)

- (a) Find the total differential of Z for the following functions:

(i) $Z=f(X, Y)=5X^3+3Y^2, Y=4X_2$ (4 marks)

(ii) $Z=f(X, Y)=6X^2+4Y^2, X=3U, Y=2U$ (4 marks)

- (b) Find the derivatives of the functions:

(i) $X^2+XY+Y^5=3$ (6 marks)

(ii) $Y=a^{x^4}$ (6 marks)

QUESTION THREE (20 MARKS)

Consider the following function:

$$Z=f(X, Y)=-X^2+XY-2Y^2+6X+4Y$$

- (a) Find the critical values of X and Y and the stationary point. (8 marks)

- (b) By applying the second order condition, determine whether the stationary point presents a maximum or a minimum. (6 marks)

QUESTION FOUR (20 MARKS)

A firm in a perfectly competitive market has the following demand, total variable costs and total fixed costs functions:

Demand function: $P=12.1$

$$\text{Total Variable Costs} = \frac{1}{20}Q^2 - 1.5Q^2 + 17.5Q$$

Total Fixed Cost = 50

- (a) Obtain the functions for;

(i) Total Cost (3 marks)

(ii) Total Revenue (3 marks)

(iii) Profits (4 marks)

- (b) Find the output level at which profits are maximized. Check the second derivate test.

(5 marks)

- (c) Compare the marginal cost and marginal revenue at the profit maximizing output and comment. (5marks)

QUESTION FIVE (20 MARKS)

- (a) Find the third degree Taylor's series of the function: $f(x) = \frac{1}{x-5}$ at $x=1$ (5 marks)

- (b) Obtain the second Maclurian series of the function $f(x) = 5 - 2x + 6x^2$. (5 marks)

- (c) Given the function $z = 7 - x^2 - y^4$, examine the stationary point of the function at $x=7$

(5 marks)

QUESTION SIX (20 MARKS)

- (a) Find the second ordered partial derivative of $Q = 4X^2 - 16XY + Y^2$ and present the derivatives in a matrix format, denote the matrix by "H". (6 marks)

- (b) Find the discriminant and the Hessian corresponding to the following quadratic form:

$$Q = f(x, y) = -4x^2 + 6xy - 5y^2 \quad (8 \text{ marks})$$

- (c) Demonstrate Euler's Theorem for $Q = AK^{\frac{2}{5}}L^{\frac{8}{5}}$ (6 marks)