

## UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATIONS FOR THE AWARD OF MASTER OF SCIENCE IN ECONOMICS

## MSEC 802: ADVANCED MICROECONOMICS

STREAMS: MSC (ECON) Y1S1
TIME: 3 HOURS
DAY/DATE: WEDNESDAY 08/8/2018
2.30 P.M. - 5.30 P.M.

INSTRUCTIONS: Answer question ONE and any other TWO questions

## QUESTION ONE

(a) Explain with reference to a specific utility function, the concept of an indirect utility function. State the properties of an indirect utility function.
(b) An individual consumer is faced with a utility function of the form:

$$
U\left(X_{1}, X_{2}\right)=4 X^{2}, X_{2}
$$

Where $\quad X_{1}$ and $X_{2}$ are commodities in his bundle, with a unit price $P_{1}$ and
$P_{2}$ respectively. If the consumers money income is "M":
(i) Derive his Marshallian demand function for the commodities $X_{1}$ and $X_{2}$
[6 marks]
(ii) Derive the relevant indirect utility function for this consumer
(iii) What are the compensated demand functions for the respective commodities
marks]
QUESTION TWO
(a) A firm aims at producing a target level of output of "Q" units. It employs two factor inputs $X_{1}$ and $X_{2}$ whose prices are $r_{1}$ and $r_{2}$ per unit respectively. Given a cost outlay of "C" outline the firms process of cost minimization. Show that for a minimum cost, the bordered Hessian determinant is negative definite marks]
(b) Assume that the firm in (a) above has a production function given by:

$$
Q\left(X_{1}, X_{2}\right)=\left(X_{1}^{\frac{1}{2}}+3 X_{2}^{\frac{1}{2}}\right)^{2}
$$

and that the prices of factor inputs are $r_{1}=r_{2}=1$. Find the cheapest way to produce 16 units of output. Estimate the respective amounts of factor inputs.
[8 marks]

## QUESTION THREE

(a) What are homothetic preferences. Illustrate and explain why perfect substitutes utility function are an example of homothetic preferences.
(b) Using Slutsky method decompose the demand change resulting from a price into two separate effects.
(c) Distinguish the two concepts of an expansion path and a price consumption time
marks]

## QUESTION FOUR

(a) A Cobb-Douglas production functions given by the expression

$$
Q=L^{\alpha} K^{\beta} \quad 0<\beta<1, \alpha+\beta=1
$$

(i) Show that elasticities of output due to labour and capital are given by ${ }^{\alpha}$ and
(ii) Show that the marginal rate of technical substitution depends only on the ratio of $\frac{K}{L}$ but on the scale of production and that it diminishes as $\quad \frac{L}{K} \quad$ increases
[5 marks]
(b) A firm produces good q using the factors of production. Capital (K) and labour (L). the price of capital is " $r$ " and the price of labour is " $W$ ". the price of output " $q$ " is " $p$ ". The production function of the firm is given by:

$$
q=\sqrt{K}+\sqrt{L}
$$

(i) Obtain the factor demand functions of K and L that will minimize the cost of producing " $q$ " unit of output.
(ii) If the price of capital is $r=1$ and the price of labour is $w=1$. What is the cost function for the firm
(iii) Obtain the firms average cost

## QUESTION FIVE

(a) Consider the following utility functions for two goods X and Y :

$$
\begin{equation*}
\cup(X, Y)=X Y \tag{i}
\end{equation*}
$$

$$
\begin{equation*}
\cup(X, Y)=X^{2} Y^{2} \tag{ii}
\end{equation*}
$$

$$
\begin{equation*}
\cup(X, Y)=\operatorname{Ln} X+\operatorname{Ln} Y \tag{iii}
\end{equation*}
$$

Show that each of these has a diminishing MRS, but they exhibit constant, increasing and decreasing marginal utility respectively
(b) A consumer derives utility from commodity X and Y according to the following functions

$$
\cup(X, Y)=\sqrt{\square}\left(X^{2}+Y^{2}\right) \text { if } P_{x}=3 \quad \text { and } \quad P_{y}=4 \quad \text { and } \mathrm{M}=50
$$

(i) Maximize the utility of this consumer

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(ii) Find the values of $q_{x}$ and $q_{y}$ demanded marks]

