

CHUKA



UNIVERSITY

## UNIVERSITY EXAMINATIONS

**EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND STATISTICS**

**ECON 234: MATHEMATICS FOR ECONOMIST II**

**STREAMS: Y2S2**

**TIME: 3 HOURS**

**DAY/DATE: TUESDAY 10/04/2018**

**2.30 P.M – 5.30 P.M**

**INSTRUCTION:**

- **Answer question one and any other three questions from the remaining**

1. The production function of a firm is given by:

$$Q = 80 + K^{1/2} + L^{1/2}$$

The corresponding constraints is given by

$$C = 2L + 2K$$

If the firm wishes to minimize its costs for a specified level of output of  $Q_0=88$

- Write out the firms constrained cost minimization problem. [3marks]
- Construct the corresponding langrangian function. [4marks]
- What are the critical values of  $\bar{L}$  and  $\bar{K}$  the function. [10marks]
- Establish the second order condition for the minimum. [10marks]
- What is the minimum cost. [3marks]

2. You are given the following polynomial

$$Z = f(x_1, x_2, x_3) = 2x_1^2 + 3x_1x_2 - 2x_1x_3 + 3x_2x_3 + x_2^2 + 5x_3^2$$

- What type of form is Z? [1mark]
- Find the Hessian matrix (4) for z? [6marks]
- Find the leading principal submatrices of H. [2marks]

(iv) Find the leading minors of H. [3marks]

(b) Determine the signs of the following quadratic forms:

(i)  $Q = 7X^2 - 3Y^2$  [2marks]

(ii)  $Q = -3X^2 + 4XY + 4Y^2$  [2marks]

(iii)  $Q = F(X,Y) = 3X^2 - 6XY - 4Y^2$  [2marks]

(iv)  $Q = f(X,Y) = -4X^2 + 4XY - 6Y^2$  [2marks]

Use the Hessian approach in part (iii) and (iv)

3. find the derivatives of the following:

(i)  $Y = \log_a(3x^2 + 7x)$   $a > 0$ ;  $a \neq 1$  [2marks]

(ii)  $Y = \log_3 \sqrt{x^4 + x^2}$  [2marks]

(iii)  $Y = \log_2(x^3 - x)$  [2marks]

(iv)  $Y = 5^{3x}$  [2marks]

(b) The growing value of some asset is given by the exponential function given below. Find the rate of growth of the value of the asset. [10marks]

$V = Be^{rt}$  where:

B – constant

r- rate of growth we wish to derive

t – time

4. (a) Evaluate the following integrals by substitution.

(i)  $\int (5x^4 + 2x)(x^5 - x^2 + 3) dx$  [3marks]

(ii)  $\int (12x^3 + 6x^2 + 1)(3x^4 + 2x^3 + x) dx$  [3marks]

(b) Compute the following integration by parts:

(i)  $\int (7x[9 + 5x]^3) dx$  [3marks]

(ii)  $\int \frac{8x}{(x-5)^5} dx$  [3marks]

(iii)  $\int 12x e^{4x} dx$  [3marks]

(c) Given the following marginal revenues functions derive the corresponding total revenue functions.

- (i)  $MR = 25 - BQ^2$        $TR = 0$  when  $Q = 0$       [2marks]  
(ii)  $MR = 50 - 4Q$        $TR = 0$  when  $Q = 0$       [3marks]

5. (a) Consider the following differential equation and the accompanying initial conditions.

$$\frac{dy}{dt} + ay = 6 \quad t = 0 \text{ and } y = y(0)$$

Find :

- (i) The homogeneous solution ( $y_n$ )      [2marks]  
(ii) The particular solution ( $y_p$ )      [2marks]  
(iii) The general solution      [2marks]  
(iv) The unique or definite solution.      [2marks]

(b) Find the second degree maclaurin series for the following functions at  $x = 0$ .

(i)  $f(x) = \frac{1}{(x-2)^2}$       [6marks]

(ii)  $f(x) = 5 - 2x + 6x^2$       [6marks]

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