CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD MASTER OF SCIENCE IN PHYSICS

PHYS 811: MATHEMATICS PHYSICS

STREAMS: MSC (PHYS) SB **TIME: 3 HOURS**

DAY/DATE: TUESDAY 05/12/2017 **11.30 A.M. – 2.30 P.M.**

INSTRUCTIONS:

- **Answer any four questions**
- **Do not write anything on the question paper**
- **This is aclosed book exam, no reference materials are allowed in the examination room**
- **There will be no use of mobile phones or any other unauthorized materials**
- **Write your answers legibly and use your time wisely**

QUESTION ONE

(a) Show that $\frac{\vec{r}}{r^2}$ is irrotational [6 marks]

(b) If $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$, show that $\frac{1}{r}$ is a solution of Laplace's equation. [6 marks]

(c) Find the value of a if the vector,
$\vec{V} = (x + 3y)\hat{\imath} + (y - 2z)\hat{\jmath} + (x + az)\hat{k}$ is solenoidal. [3 marks]

QUESTION TWO

(a) Find the Eigen values and normalized vector of the matrix. [5 marks]

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(b) Use matrix A to verify Cayley-Hamilton theorem. Hence find A^{-1}. [10 marks]

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{bmatrix}$$

QUESTION THREE

(a) Evaluate $\int_0^\infty x^7 e^{-x}\, dx$ [3 marks]

(b) Evaluate $I = \int_0^1 x^5 (1-x)^4\, dx$ [3 marks]

(c) Solve the differential equation, $\frac{d^2\psi}{dx^2} + (E - x^2)\,\psi = 0$ such that $\psi \to 0$ as $|x| \longrightarrow \infty$.
You may need to put $\psi = V \frac{-x^2}{e^2}$ [9 marks]

QUESTION FOUR

(a) Using Fourier'sseries prove that $\sum_{n=1}^{\infty} \frac{1}{n^6} = \frac{\pi^2}{6}$ [8 marks]

(b) Find the finite Fourier cosine transform of x. [7 marks]

QUESTION FIVE

(a) Find the steady temperature distribution of a thin rectangular plate bounded by the lines $x = 0, x = l, y = 0\ andy = b$, assuming that the edges $x = 0, x = 1$ and $y = 0$ are being kept at a temperature zero while the edge $y = b$ maintained at temperature $F\,(x)$.
[10 marks]

(b) Find the Laplace transform of the following functions

$(i) F\,(t) = 1$

$(ii) F\,(t) = t$

$(iii) F\,(t) = e^{at}$ [5 marks]
