CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF SCIENCE IN MATHEMATICS (BIOSTATISTICS)

MATH 842: MEASURE THEORY AND PROBABILITY

STREAMS: **TIME: 3 HOURS**

DAY/DATE: THURSDAY 14/12/2017 **2.30 P.M – 5.30 P.M**

INSTRUCTIONS:

- **Answer any three questions**
- **Do not write on the question paper**

QUESTION ONE: (20 MARKS)

a) Define the following terms:

i) σ-algebra [2 marks]

ii) Borelσ-algebra [2 marks]

iii) Measurable set [1 mark]

b) Let $\{\varepsilon_i : i \in I\}$ be a collection of σ-algebras. Show that how that $\bigcap_{i\in I} \varepsilon_i$ is also a σ-algebras.[7 marks]

c) Let $\mathcal{B} = \mathcal{B}(\mathbb{R})$ be the borel algebra of $\mathbb{R}$, R be the ring of half-open intervals and $I = \{(a, b]: a < b\}$ be the set of all half-open intervals. Show that $\sigma(R) = \sigma(I) = \mathcal{B}$. [8 marks]

QUESTION TWO: (20 MARKS)

a) Define the following terms:

i) Set function [1 mark]

ii) Probability measure [4 marks]

iii) Continuous set function [5 marks]

b) Let ε be a ring on E and $\mu: \varepsilon \longrightarrow [0, \infty]$ be an additive set function. Show that if μ is countably additive, then μ is continuous at all $A \in \varepsilon$. [10 marks]

QUESTION THREE: (20 MARKS)

a) Prove that there exists a unique Borel measure μ on $(\mathbb{R}, \mathcal{B})$ such that $\mu\big((a, b]\big) = b - a$ for all $a, b \in \mathbb{R}$with $a < b$. [10 marks]

b) (i) Describe an independent σ-algebra. [4 marks]

(ii) Let $\mathcal{E}_1, \mathcal{E}_2 \in \mathcal{E}$ be π-systems and suppose that $\mathbb{P}(A_1 \cap A_2) = \mathbb{P}(A_1)\mathbb{P}(A_2)$whenever $A_1 \in \varepsilon_1$, $A_2 \in \varepsilon_2$. Show that $\sigma(\varepsilon_1)$ and $\sigma(\varepsilon_2)$ are independent. [6 marks]

QUESTION FOUR (20 MARKS)

a) Describe the following terms as used in measure theory

i) Measurable function [2 marks]

ii) ε-simple function [4 marks]

iii) Distribution function [4 marks]

iv) Almost everywhere convergence [3 marks]

b) Prove the Kolmogorov's 0-1 law, that is, if $\{x_n\}$ is a sequence of independence variables, then every tail event has probability 0 or 1, and any T- measureable random variable Y is almost surely constant. [7 marks]
