CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF SCIENCE IN MATHEMATICS (BIOSTATISTICS)

MATH 812: FIELD THEORY

STREAMS: **TIME: 3 HOURS**

DAY/DATE: THURSDAY 14/12/2017 **8.30 A.M – 11.30 A.M**

INSTRUCTIONS:

- **Answer any three questions**
- **Do not write on the question paper**

QUESTION ONE: (20 MARKS)

a) Define the following terms:

i) Ring [4 marks]

ii) Ideal [2 marks]

iii) Field [2 mark]

b) Let I be an ideal in a ring R. Show that:

i) R/I is a ring under coset addition and coset multiplication. [10 marks]

ii) R/I is commutative if R is commurtative [1 mark]

iii) R/I has a unit element if R has [1 mark]

QUESTION TWO: (20 MARKS)

a) Show that every field is an integral domain [6 marks]

b) Show that every finite integral domain is a field [8 marks]

c) Show that multiplication cancellation laws hold in a ring R if and only if R has no divisors of zero. [6 marks]

QUESTION THREE: (20 MARKS)

a) Describe the following terms as used in field theory:

i) Algebraic element [4 marks]

ii) Splitting field [4 marks]

iii) Algebraically closed field extension [2 marks]

b) Let K/F be a field extension and $\alpha \in K$. Show that α is algebraic over F if $[F(\alpha):F]$ is finite. [4 marks]

c) Let K/F be an extension of fields and f an irreducible polynomial in $F[t]$. Show that if $\alpha \in K$ is a root of f, then there exists an F-isomorphism $\theta: F[t]/(f) \longrightarrow F(\alpha)$ given by $1 + (f) \mapsto \alpha$. [6 marks]

QUESTION FOUR (20 MARKS)

a) Describe the following terms as used in Galois Theory:

i) Dependent Character [4 marks]

ii) Galois extension [3 marks]

iii) Conjugate field extension [4 marks]

b) Let K/F be a finite extension of fields. Show that the Galois group $G(K/F)$ is a finite group and satisfies $[K:F] \geq |G(K/F)|$. [4 marks]

c) Let K/F be a finite extension of fields. Show that it is also Galois if $[K:F] = |G(K/F)|$. [5 marks]
