CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS
EMBU CAMPUS

EXAMINATION FOR THE AWARD OF DIPLOMA IN EDUCATION (ARTS)

MATH 0224: PROBABILITY AND STATISTICS II

STREAMS: **TIME: 2 HOURS**

DAY/DATE:

INSTRUCTIONS:

- **Answer question one and any other two questions**

1. (a) Define the following terms; [4marks]

(i) Random variable

(ii) Discrete variable

(iii) Continuous variable

(iv) Sample space

(b) Let X_1 *and* X_2 have a joint probability distribution function

$$F(x_1, x_2) = \begin{cases} x_1 + x_2 \, 0 < 1 \ , & 0 < \ x_1 < 1, \quad 0 < x_2 \ < 1 \\ 0 & elsewhere \end{cases}$$

Obtain the following

(i) Marginal probability density of X_1 *and* X_2 [4marks]

(ii) Conditional p.d.f of $X_{1,}$given $X_2 = x_2$[3marks]

(iii) Probability of pr$(X_1 + X_2 \leq 1)$ [3marks]

(c) Given the probability distribution function of a Poisson distribution as

$$F(x) = \begin{cases} \frac{\lambda^x e^{-\lambda}}{x!}, & x = 0,1,2,3 \ldots \\ 0, & elsewhere \end{cases}$$

Obtain

(i) Moment generating function [3marks]

(ii) Mean and variance [5marks]

(d) A discrete random variable has the following probability mass function

X	3	4	5	6	7
P(X=x)	0.1	a	0.3	b	0.2

If E[X] = 5.2

(a) Find the value of a and b [4marks]

(b) The mean and variance of X [4marks]

2. (a) A certain couple was planning to have 7 children. Their chance of having a boy child is 0.8. What is the chance of having.

(i) Exactly 2 boys [4marks]

(ii) At least 2 boys [4marks]

(b)If g(x) is a function of the random variable X defined by g(x) = a+bx, where a and b are constants. Show that the variance of the function g(x) is given by Var[g(x)] = b^2var (x). [4marks]

(c) A fair die is thrown once. Find the 1st , 2nd and the 3rd factorial moments. [8marks]

3. (a) A CDF is given by $f(x) = \begin{cases} 1 - e^{-2x}, & x > 0 \\ 0 & elsewhere \end{cases}$

(i) Derive the pdf . [3marks]

(ii) Show that the derived function is a pdf. [5marks]

(iii) Find P(2$\leq x \leq$ 3) [5marks]

(b)The random variable X has a probability mass function.

$$P(X = x) = \begin{cases} \frac{x}{10} & x = 1,2,3,4 \\ 0, & elsewhere \end{cases}$$

(a) Compute E[$5X^3$-$2X^2$] [7marks]

4. (a) If the function is a probability density function.

$$F(x) = \begin{cases} \frac{2}{39}\ (1+x) & 4 \le x \le 7 \\ 0\ , & otherwise \end{cases}$$

Calculate the probability that

(i) Pr (x < 5) [3marks]

(ii) Pr (5$\le x \le 6.5$) [4marks]

(b) A continuous random variable X is said t have a normal distribution if the probability distribution function is given by

$$F(x) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma}} e & -\frac{1}{2}\left(\frac{(x-\mu)^2}{\sigma^2}\right) \\ 0, & elsewhere \end{cases}$$

(g) Derive the moment generating function and hence obtain the mean and variance of x. [13marks]
