

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

**SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR
OF APPLIED COMPUTER SCIENCE**

ACSC 271/ACMP 203: MATHEMATICAL METHODS FOR COMPUTER SCIENCE

STREAMS: B.Sc (ACSC)

TIME: 2 HOURS

DAY/DATE: THURSDAY 7/12/2017

8.30 A.M - 10.30 A.M.

INSTRUCTIONS:

- Answer Question ONE (COMPULSORY) and any other TWO Questions.
- Adhere to the instructions on the answer booklet.

QUESTION ONE [30 MARKS](a) Obtain the domain of the function $f(x) = \frac{1}{x^2-4}$ [3 Marks](b) Find the limit of the function $f(x) = \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2-1}$ as $x \rightarrow 1$ [4 Marks](c) Obtain the gradient of the function $f(x) = \frac{2}{x}$ from first principles. [4 Marks](d) Find the gradient of the function $f(x) = y = 2^x$ at $x = 0$ [4 Marks](e) Obtain $\frac{dy}{dx}$ for the function $y = \sin^{-1}(3x + 1)$ [4 Marks](f) Solve the differential equation $\frac{dy}{dx} = \frac{x}{x+1}$ [4 Marks]

(g) Solve the differential equation below

$$\begin{array}{l} x + 2y = 1 \\ -2x - 3y = 2 \end{array} \quad \text{by row reduction} \quad [3 \text{ Marks}]$$

(h) Find the angle between the vectors $\vec{a} = 3i - 2j + 4k$ and $\vec{b} = -4i + 3j - 2k$ [4 Marks]

QUESTION TWO

(a) Obtain the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ [11 Marks]

(b) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ is convergent using the ratio test. [4 Marks]

(c) Using the ratio test, prove that the series $\sum_{n=1}^{\infty} \frac{2^n}{n^4}$ is divergent. [5 Marks]

QUESTION THREE

(a) Calculate the area of the triangle PQR with vertices $P(1, 2, 0)$ $Q(3, 0, -3)$ and $R(5, 2, 6)$ by the cross product. [5 Marks]

(b) Find the value of t for which the vectors $a = ti - 5j + 2k$ and $b = i + 4j - tk$ are orthogonal.

(c) Test whether the differential equation $(e^y + 1) \cos x dx + e^y \sin x dy = 0$ is exact hence solve it. [5 Marks]

(d) Solve the system below by row reduction

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 1 \\ -x_1 + 2x_2 - 3x_3 &= 3 \\ x_1 + 3x_2 - 2x_3 &= 4 \end{aligned}$$

[5 Marks]

(e) Solve the differential equation $\frac{dy}{dx} = 3x^2 + 5$ given that $y(0) = 10$

QUESTION FOUR

(a) Find the inverse of the matrix A by the Cayley Hamilton theorem

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

[8 Marks]

(b) Approximate $\int_1^2 \frac{dx}{x}$ by the mid point rule with 5 strips and obtain the actual error correct to 4 decimal points. [8 Marks]

(c) Obtain the derivative $\frac{dy}{dx}$ of the function $y^2 + x^5 = 2$ [4 Marks]

QUESTION FIVE

(a) Find the domain of the function $f(x) = \sqrt{x^2 - x - 6}$ [4 Marks]

(b) Evaluate $\lim_{x \rightarrow \infty} \frac{\sqrt[3]{x^2+3}}{\sqrt{27x^2-1}}$ [3 Marks]

(c) Evaluate $\frac{dy}{dx}$ of $f(x) = \sqrt[3]{3x^3 + 7x}$ [3 Marks]

(d) Find the equation of the normal to the curve $y = x^2 - 6x + 5$ at the point $x = 1$ [4 Marks]

(e) Evaluate $\frac{dy}{dx}$ of $\tan^{-1}(\ln x)$ [3 Marks]

(f) Find the area of the parallelogram spanned by the given vectors $\vec{a} = i + 2j - k$, and $\vec{b} = 2i + 3j + k$ [3 Marks]

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